LESSON MAKER PRO

Factor Beyond Real

Use complex numbers to factor sums of squares.

HIGH SCHOOL · HSN-CN.C.8

CCSS.Math.Content.HSN-CN.C.8 "(+) Extend polynomial identities to the complex numbers. For example, rewrite x 2 + 4 as (x + 2i)(x - 2i)."

NAME _____ DATE _____

COMPLEX FACTOR PAIRS

Because i^2 = -1, a sum of squares can be written as a difference of squares. Use a^2 + b^2 = (a + bi)(a - bi). The two factors are conjugates, so their imaginary middle terms cancel when multiplied.

WORKED EXAMPLE

Factor x^2 + 49 completely over the complex numbers.

1. i^2 = -1, so (7i)^2 = -49.
2. Rewrite x^2 + 49 as x^2 - (7i)^2.
3. Apply the difference of squares pattern.
4. x^2 + 49 = (x + 7i)(x - 7i).

Answer: (x + 7i)(x - 7i)

PRACTICE 6 problems

1 Factor x^2 + 9 completely over the complex numbers.

ANSWER _____

2 Factor x^2 + 36 completely over the complex numbers.

ANSWER _____

3 Factor 4x^2 + 25 completely over the complex numbers.

ANSWER _____

4 Factor 9x^2 + 4 completely over the complex numbers.

ANSWER _____

5 Factor 16x^2 + 9 completely over the complex numbers.

ANSWER _____

6 Expand and simplify (x + 8i)(x - 8i).

ANSWER _____

APPLY IT Show your work

1 A student who runs a music podcast is coding a simple audio filter. Its test equation is f(x) = x^2 + 4. Write f(x) in complex linear factors, then list its complex zeros.

WORK SPACE

ANSWER _____

2 A robotics team is testing a virtual motor model with g(x) = 4x^2 + 1. The simulation needs the expression in complex linear factors and needs its complex zeros.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Factor x^4 + 10x^2 + 25 completely over the complex numbers. Show the perfect-square step before writing the complex linear factors.

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ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Factor x^2 + 9 completely over the complex numbers.	**(x + 3i)(x - 3i)**
2	Factor x^2 + 36 completely over the complex numbers.	**(x + 6i)(x - 6i)**
3	Factor 4x^2 + 25 completely over the complex numbers.	**(2x + 5i)(2x - 5i)**
4	Factor 9x^2 + 4 completely over the complex numbers.	**(3x + 2i)(3x - 2i)**
5	Factor 16x^2 + 9 completely over the complex numbers.	**(4x + 3i)(4x - 3i)**
6	Expand and simplify (x + 8i)(x - 8i).	**x^2 + 64**

PART 2 · APPLY IT

1. **f(x) = (x + 2i)(x - 2i); the zeros are x = 2i and x = -2i.** Rewrite x^2 + 4 as x^2 - (2i)^2, then factor as a difference of squares. Set each linear factor equal to zero to find the zeros.
2. **g(x) = (2x + i)(2x - i); the zeros are x = i/2 and x = -i/2.** Rewrite 4x^2 + 1 as (2x)^2 - i^2 and factor the difference of squares. Solve each factor equal to zero for x.

CHALLENGE

x^4 + 10x^2 + 25 = (x^2 + 5)^2 = (x + sqrt(5)i)^2(x - sqrt(5)i)^2

Accept conjugate factors in either order. For the challenge, accept an equivalent answer with the repeated factors written separately instead of using exponents.