LESSON MAKER PRO

Factor Beyond Real

Use complex numbers to factor sums of squares.

HIGH SCHOOL · HSN-CN.C.8

CCSS.Math.Content.HSN-CN.C.8 "(+) Extend polynomial identities to the complex numbers. For example, rewrite x 2 + 4 as (x + 2i)(x - 2i)."

NAME _____ DATE _____

COMPLEX FACTOR PAIRS

Because $i^2 = -1$, a sum of squares can be written as a difference of squares. Use $a^2 + b^2 = (a + bi)(a - bi)$. The two factors are conjugates, so their imaginary middle terms cancel when multiplied.

WORKED EXAMPLE

Factor $x^2 + 49$ completely over the complex numbers.

1. $i^2 = -1$, so $(7i)^2 = -49$.
2. Rewrite $x^2 + 49$ as $x^2 - (7i)^2$.
3. Apply the difference of squares pattern.
4. $x^2 + 49 = (x + 7i)(x - 7i)$.

Answer: (x + 7i)(x - 7i)

PRACTICE 6 problems

1 Factor $x^2 + 9$ completely over the complex numbers.

ANSWER _____

2 Factor $x^2 + 36$ completely over the complex numbers.

ANSWER _____

3 Factor $4x^2 + 25$ completely over the complex numbers.

ANSWER _____

4 Factor $9x^2 + 4$ completely over the complex numbers.

ANSWER _____

5 Factor $16x^2 + 9$ completely over the complex numbers.

ANSWER _____

6 Expand and simplify $(x + 8i)(x - 8i)$.

ANSWER _____

APPLY IT Show your work

1 A student who runs a music podcast is coding a simple audio filter. Its test equation is $f(x) = x^2 + 4$. Write f(x) in complex linear factors, then list its complex zeros.

WORK SPACE

ANSWER _____

2 A robotics team is testing a virtual motor model with $g(x) = 4x^2 + 1$. The simulation needs the expression in complex linear factors and needs its complex zeros.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Factor $x^4 + 10x^2 + 25$ completely over the complex numbers. Show the perfect-square step before writing the complex linear factors.

WORK SPACE AND ANSWER

ANSWER KEY

Factor Beyond Real

HIGH SCHOOL · HSN-CN.C.8

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Factor $x^2 + 9$ completely over the complex numbers.	**$(x + 3i)(x - 3i)$**
2	Factor $x^2 + 36$ completely over the complex numbers.	**$(x + 6i)(x - 6i)$**
3	Factor $4x^2 + 25$ completely over the complex numbers.	**$(2x + 5i)(2x - 5i)$**
4	Factor $9x^2 + 4$ completely over the complex numbers.	**$(3x + 2i)(3x - 2i)$**
5	Factor $16x^2 + 9$ completely over the complex numbers.	**$(4x + 3i)(4x - 3i)$**
6	Expand and simplify $(x + 8i)(x - 8i)$.	**$x^2 + 64$**

PART 2 · APPLY IT

1. **f(x) = (x + 2i)(x - 2i); the zeros are x = 2i and x = -2i.** Rewrite $x^2 + 4$ as $x^2 - (2i)^2$, then factor as a difference of squares. Set each linear factor equal to zero to find the zeros.
2. **g(x) = (2x + i)(2x - i); the zeros are x = i/2 and x = -i/2.** Rewrite $4x^2 + 1$ as $(2x)^2 - i^2$ and factor the difference of squares. Solve each factor equal to zero for x.

CHALLENGE

$x^4 + 10x^2 + 25 = (x^2 + 5)^2 = (x + \text{sqrt}(5)i)^2(x - \text{sqrt}(5)i)^2$

Accept conjugate factors in either order. For the challenge, accept an equivalent answer with the repeated factors written separately instead of using exponents.