LESSON MAKER PRO

Roots Beyond Real

Find every complex root, including repeated roots.

HIGH SCHOOL · HSN-CN.C.9

CCSS.Math.Content.HSN-CN.C.9 "(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials."

NAME ______________________________ DATE ______________

TWO ROOTS, COUNTED CAREFULLY

The Fundamental Theorem of Algebra says that a polynomial of degree n has exactly n complex roots when repeated roots are counted. For a quadratic, this means there are always two complex roots. They may be two different real roots, two nonreal complex roots, or one repeated real root.

WORKED EXAMPLE

Solve x^2 - 7x + 10 = 0.

1. Factor: x^2 - 7x + 10 = (x - 2)(x - 5).
2. Set each factor equal to 0: x - 2 = 0 or x - 5 = 0.
3. x = 2 or x = 5.

Answer: x = 2, 5

PRACTICE

6 problems

1 Solve x^2 + 1 = 0.

ANSWER ______________________________

2 Solve x^2 - 8x + 17 = 0.

ANSWER ______________________________

3 Solve x^2 + 6x + 18 = 0.

ANSWER ______________________________

4 Find all roots of x^2 - 12x + 36 = 0. State the multiplicity.

ANSWER ______________________________

5 Solve x^2 + 16x + 65 = 0.

ANSWER ______________________________

6 Solve x^2 - 9 = 0.

ANSWER ______________________________

APPLY IT
Show your work

1 A game designer models an avatar's height above the ground by $h = -t^2 + 8t + 33$, where t is the number of seconds after launch. When is the avatar at ground level? Give the algebraic roots, then identify the time that makes sense in the situation.

WORK SPACE

ANSWER ____________________

2 An audio filter in a music app is tuned by the characteristic equation $z^2 + 12z + 45 = 0$. Find both complex zeros of the filter.

WORK SPACE

ANSWER ____________________

CHALLENGE
One step up

CHALLENGE

Find all complex roots of $x^4 + 14x^2 + 33 = 0$. Then explain how the number of roots supports the Fundamental Theorem of Algebra.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $x^2 + 1 = 0$.	**$x = i, -i$**
2	Solve $x^2 - 8x + 17 = 0$.	**$x = 4 + i, 4 - i$**
3	Solve $x^2 + 6x + 18 = 0$.	**$x = -3 + 3i, -3 - 3i$**
4	Find all roots of $x^2 - 12x + 36 = 0$. State the multiplicity.	**$x = 6$ with multiplicity 2**
5	Solve $x^2 + 16x + 65 = 0$.	**$x = -8 + i, -8 - i$**
6	Solve $x^2 - 9 = 0$.	**$x = 3, -3$**

PART 2 · APPLY IT

1. **Algebraic roots: $t = -3$ and $t = 11$. Meaningful time: $t = 11$ seconds.** Set h equal to 0 and factor: $-t^2 + 8t + 33 = -(t - 11)(t + 3)$. The algebraic roots are -3 and 11, but only 11 is at or after launch.
2. **$z = -6 + 3i, -6 - 3i$** The discriminant is $144 - 180 = -36$, so $z = (-12 \pm 6i)/2 = -6 \pm 3i$.

CHALLENGE

$x^4 + 14x^2 + 33 = (x^2 + 3)(x^2 + 11)$. The roots are $x = i\sqrt{3}, -i\sqrt{3}, i\sqrt{11}, -i\sqrt{11}$. There are four complex roots, which matches the polynomial's degree of 4.

Accept roots in any order and equivalent forms such as 4 + i and 4 - i. For the repeated-root problem, require the multiplicity statement; for the height context, accept 11 seconds as the physical answer while recognizing -3 as an algebraic root.