LESSON MAKER PRO

Roots Beyond Real

Use complex numbers to find every quadratic root.

HIGH SCHOOL · HSN-CN.C.9

CCSS.Math.Content.HSN-CN.C.9 "(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials."

NAME _____ DATE _____

EVERY QUADRATIC HAS ROOTS

The Fundamental Theorem of Algebra says that a polynomial of degree n has exactly n complex roots when repeated roots are counted. For a quadratic $ax^2 + bx + c$, the quadratic formula always gives two roots, which may be real, repeated, or complex conjugates.

WORKED EXAMPLE

Solve $x^2 - 6x + 25 = 0$.

1. Move the constant form into a square: $(x - 3)^2 + 16 = 0$.
2. Subtract 16: $(x - 3)^2 = -16$.
3. Take square roots: $x - 3 = \pm 4i$.
4. Add 3: $x = 3 \pm 4i$.

Answer: x = 3 + 4i and x = 3 - 4i

PRACTICE 6 problems

1 Solve $x^2 + 9x + 14 = 0$.

ANSWER _____

2 Solve $x^2 - 12x + 35 = 0$.

ANSWER _____

3 Solve $x^2 + 10x + 29 = 0$.

ANSWER _____

4 Solve $x^2 + 14x + 49 = 0$. State the root and its multiplicity.

ANSWER _____

5 Use the quadratic formula to solve $x^2 - x + 17 = 0$.

ANSWER _____

6 Solve $x^2 + 81 = 0$. Then state how many complex roots the polynomial has, counting multiplicity.

ANSWER _____

APPLY IT Show your work

1 A drone club models the height of a practice drone, in meters, by $h(t) = -7t^2 + 56t + 63$, where t is time in seconds. Find both algebraic times when $h(t) = 0$. Which time tells when the drone reaches the ground after launch?

WORK SPACE

ANSWER _____

2 For a student concert, organizers model net balance B(x), in hundreds of dollars, by $B(x) = x^2 - 10x + 29$, where x is the ticket price in dollars. A balance of 0 would mean breaking even. Find the complex roots, then decide whether any real ticket price gives a break-even balance.

WORK SPACE

ANSWER _____

CHALLENGE One step up

In 3 sentences, explain why every quadratic $ax^2 + bx + c = 0$, where a is not 0, has exactly 2 complex roots when multiplicity is counted. Use the quadratic formula and address what happens when $b^2 - 4ac$ is positive, zero, or negative.

WORK SPACE AND ANSWER

Roots Beyond Real

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $x^2 + 9x + 14 = 0$.	**x = -7 and x = -2**
2	Solve $x^2 - 12x + 35 = 0$.	**x = 5 and x = 7**
3	Solve $x^2 + 10x + 29 = 0$.	**x = -5 + 2i and x = -5 - 2i**
4	Solve $x^2 + 14x + 49 = 0$. State the root and its multiplicity.	**x = -7, multiplicity 2**
5	Use the quadratic formula to solve $x^2 - x + 17 = 0$.	**x = (1 + i√67)/2 and x = (1 - i√67)/2**
6	Solve $x^2 + 81 = 0$. Then state how many complex roots the polynomial has, counting multiplicity.	**x = 9i and x = -9i; 2 complex roots**

PART 2 · APPLY IT

1. **The algebraic times are t = -1 and t = 9. The drone reaches the ground after 9 seconds.** Set $-7t^2 + 56t + 63$ equal to 0, then divide by -7 to get $t^2 - 8t - 9 = 0$. Its roots are -1 and 9, but only the nonnegative time fits the situation.
2. **The roots are x = 5 + 2i and x = 5 - 2i. No real ticket price gives a break-even balance.** Set B(x) equal to 0 and use the quadratic formula or complete the square. Because both roots are nonreal, the model has no real x-value where B(x) = 0.

CHALLENGE

The quadratic formula gives $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$, so it produces two values when the discriminant is positive or negative. A positive discriminant gives two real roots, while a negative discriminant gives two nonreal complex conjugate roots. A zero discriminant gives one repeated real root, which counts as two roots by multiplicity.

Accept roots in either order and accept equivalent complex forms, such as -5 ± 2i. For repeated roots, students must identify that the single value counts twice.