

Vector Route Builder

Add directions, components, and diagonal results.

HIGH SCHOOL • HSN-VM.B.4a

CCSS.Math.Content.HSN-VM.B.4a "Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes."

NAME _____

DATE _____

BUILDING A VECTOR SUM

To add vectors end-to-end, place the tail of the second vector at the head of the first vector, then draw the resultant from the first tail to the final head. Add component-wise by adding horizontal components and vertical components, or place both tails together and use the diagonal of the parallelogram. The magnitude of a sum is usually not the same as the sum of the two magnitudes because vector directions can partly cancel.

WORKED EXAMPLE

Add $\langle 8, -7 \rangle + \langle -3, 4 \rangle$. Then compare $|\langle 8, -7 \rangle + \langle -3, 4 \rangle|$ with $|\langle 8, -7 \rangle| + |\langle -3, 4 \rangle|$.

1. Add horizontal components: $8 + (-3) = 5$.
2. Add vertical components: $-7 + 4 = -3$.
3. Write the sum: $\langle 8, -7 \rangle + \langle -3, 4 \rangle = \langle 5, -3 \rangle$.
4. Find magnitudes: $|\langle 5, -3 \rangle| = \sqrt{34}$, while $|\langle 8, -7 \rangle| + |\langle -3, 4 \rangle| = \sqrt{113} + 5$.

Answer: $\langle 5, -3 \rangle$; $\sqrt{34}$ is not equal to $\sqrt{113} + 5$.

PRACTICE 6 problems

- 1** Find the sum: $\langle 2, 1 \rangle + \langle 6, -9 \rangle$.

ANSWER _____

- 2** Find the sum: $\langle -10, 12 \rangle + \langle 9, 2 \rangle$.

ANSWER _____

- 3** A vector moves 11 units right and 2 units down. A second vector then moves 1 unit left and 10 units up. Write the resultant vector.

ANSWER _____

- 4** Two adjacent sides of a parallelogram are represented by $\langle 6, 9 \rangle$ and $\langle -2, 11 \rangle$. What vector represents the diagonal from their shared tail?

ANSWER _____

- 5** Let $u = \langle 12, 9 \rangle$ and $v = \langle -6, 2 \rangle$. Find $u + v$, $|u + v|$, and $|u| + |v|$. Are the last two values equal?

ANSWER _____

- 6** Find the sum: $\langle -9, 13 \rangle + \langle 11, -6 \rangle$.

ANSWER _____

APPLY IT Show your work

- 1** During a campus scavenger hunt, Jordan walks 12 blocks east and then 5 blocks north. Represent the overall displacement as a vector and find its magnitude. Is the displacement magnitude equal to the total number of blocks walked?

WORK SPACE

ANSWER _____

- 2** In a video game, an avatar moves 9 units left and 6 units up, then moves 4 units right and 14 units down. What is the avatar's net displacement vector and its magnitude?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two vectors are $u = \langle 6, 8 \rangle$ and $v = \langle 8, -6 \rangle$. Each vector has magnitude 10. Find $u + v$ and $|u + v|$. Then explain why $|u + v|$ is less than $|u| + |v|$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the sum: $\langle 2, 1 \rangle + \langle 6, -9 \rangle$.	$\langle 8, -8 \rangle$
2	Find the sum: $\langle -10, 12 \rangle + \langle 9, 2 \rangle$.	$\langle -1, 14 \rangle$
3	A vector moves 11 units right and 2 units down. A second vector then moves 1 unit left and 10 units up. Write the resultant vector.	$\langle 10, 8 \rangle$
4	Two adjacent sides of a parallelogram are represented by $\langle 6, 9 \rangle$ and $\langle -2, 11 \rangle$. What vector represents the diagonal from their shared tail?	$\langle 4, 20 \rangle$
5	Let $u = \langle 12, 9 \rangle$ and $v = \langle -6, 2 \rangle$. Find $u + v$, $ u + v $, and $ u + v $. Are the last two values equal?	$u + v = \langle 6, 11 \rangle$; $ u + v = \sqrt{157}$; $ u + v = 15 + 2\sqrt{10}$; no.
6	Find the sum: $\langle -9, 13 \rangle + \langle 11, -6 \rangle$.	$\langle 2, 7 \rangle$

PART 2 • APPLY IT

- Overall displacement: $\langle 12, 5 \rangle$. Magnitude: 13 blocks. No, Jordan walks 17 blocks total.** Use east as positive horizontal and north as positive vertical, so the sum is $\langle 12, 5 \rangle$. Its magnitude is $\sqrt{12^2 + 5^2} = 13$, which is less than $12 + 5$.
- Net displacement: $\langle -5, -8 \rangle$. Magnitude: $\sqrt{89}$ units.** Add the movement vectors $\langle -9, 6 \rangle$ and $\langle 4, -14 \rangle$ to get $\langle -5, -8 \rangle$. The magnitude is $\sqrt{(-5)^2 + (-8)^2} = \sqrt{89}$.

CHALLENGE

$u + v = \langle 14, 2 \rangle$ and $|u + v| = \sqrt{200} = 10\sqrt{2}$. Since $10\sqrt{2}$ is less than 20, the sum has a smaller magnitude than $|u| + |v|$. The vertical components partly cancel because one vector points up and the other points down.

Accept equivalent vector notation, including ordered pairs, when component order is clear. Accept $\sqrt{200}$ or $10\sqrt{2}$ for the challenge magnitude, and require an explanation of partial cancellation for the challenge.