

Matrix Power Moves

Use zero, identity, determinants, and inverses.

HIGH SCHOOL • HSN-VM.C.10

CCSS.Math.Content.HSN-VM.C.10 "(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers..."

NAME _____

DATE _____

KEY MATRIX RULES

For matrices of the same size, $A + 0 = A$, and for a compatible identity matrix, $AI = IA = A$. A square matrix has an inverse exactly when its determinant is not 0. For a 2 by 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $ad - bc$.

WORKED EXAMPLE

Find $\det(A)$ and A^{-1} for $A = \begin{bmatrix} 7 & 4 \\ 6 & 5 \end{bmatrix}$.

- $\det(A) = 7(5) - 4(6)$.
- $\det(A) = 35 - 24 = 11$.
- Because $\det(A)$ is not 0, $A^{-1} = \frac{1}{11} \begin{bmatrix} 5 & -4 \\ -6 & 7 \end{bmatrix}$.

Answer: $\det(A) = 11$; $A^{-1} = \begin{bmatrix} 5/11 & -4/11 \\ -6/11 & 7/11 \end{bmatrix}$

PRACTICE 6 problems

- 1** Let $A = \begin{bmatrix} 8 & -2 \\ 3 & 9 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Find $A + O$.

ANSWER _____

- 2** Let $B = \begin{bmatrix} -1 & 8 \\ 9 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find BI .

ANSWER _____

- 3** Find $\det(C)$ for $C = \begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix}$. Does C have an inverse?

ANSWER _____

- 4** Does $D = \begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix}$ have an inverse? Show the determinant.

ANSWER _____

- 5** Find the value of x that makes $E = \begin{bmatrix} x & 2 \\ 8 & 3 \end{bmatrix}$ have no inverse.

ANSWER _____

- 6** Find F^{-1} for $F = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.

ANSWER _____

APPLY IT Show your work

- 1** A student council models a social media campaign with $y = Mx$, where $x = \begin{bmatrix} \text{video posts} \\ \text{photo posts} \end{bmatrix}$, $y = \begin{bmatrix} \text{sign-ups in tens} \\ \text{shares in tens} \end{bmatrix}$, and $M = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$. At the end of the campaign, $y = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$. Use M^{-1} to find x .

WORK SPACE

ANSWER _____

- 2** A student coding a photo app uses the color adjustment matrix $A = \begin{bmatrix} 9 & -2 \\ 3 & 8 \end{bmatrix}$. Choosing “no adjustment” adds the zero matrix $Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Find $A + Z$ and state what it means for the photo.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For what real values of x does $G = \begin{bmatrix} x & 3 \\ 8 & x \end{bmatrix}$ have a multiplicative inverse? Explain your reasoning using the determinant.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let $A = \begin{bmatrix} 8 & -2 \\ 3 & 9 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Find $A + O$.	$\begin{bmatrix} 8 & -2 \\ 3 & 9 \end{bmatrix}$
2	Let $B = \begin{bmatrix} -1 & 8 \\ 9 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Find BI .	$\begin{bmatrix} -1 & 8 \\ 9 & 0 \end{bmatrix}$
3	Find $\det(C)$ for $C = \begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix}$. Does C have an inverse?	$\det(C) = -6$; yes, C has an inverse.
4	Does $D = \begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix}$ have an inverse? Show the determinant.	No; $\det(D) = 3(3) - 1(9) = 0$.
5	Find the value of x that makes $E = \begin{bmatrix} x & 2 \\ 8 & 3 \end{bmatrix}$ have no inverse.	$x = 16/3$
6	Find F^{-1} for $F = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$.	$F^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

PART 2 · APPLY IT

- $\mathbf{x = \begin{bmatrix} 3 \\ 2 \end{bmatrix}}$, so the campaign used **3 video posts and 2 photo posts**. Since $M^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$, multiply $M^{-1}\begin{bmatrix} 8 \\ 13 \end{bmatrix}$. The product is $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$.
- $\mathbf{A + Z = \begin{bmatrix} 9 & -2 \\ 3 & 8 \end{bmatrix}}$. The photo's color adjustment is unchanged. Adding the zero matrix leaves every entry of A unchanged. This is the matrix version of adding 0 to a number.

CHALLENGE

G has a multiplicative inverse for all real x except $x = 2\sqrt{6}$ and $x = -2\sqrt{6}$. Its determinant is $x^2 - 24$, which must not equal 0.

Accept equivalent matrix formatting and fractions written as entries or with a common factor outside the matrix. For the challenge, accept any equivalent statement that excludes both roots of $x^2 - 24 = 0$.