

Matrix Motion Lab

Use matrices to transform column vectors.

HIGH SCHOOL • HSN-VM.C.11

CCSS.Math.Content.HSN-VM.C.11 "(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors."

NAME _____

DATE _____

MATRIX AS A TRANSFORMER

A matrix transforms a column vector when you multiply the matrix on the left by the vector. Each entry in the output comes from one row of the matrix: multiply matching entries, then add the products. The number of columns in the matrix must match the number of entries in the vector.

WORKED EXAMPLE

Let $A = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$ and $v = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$. Find Av .

- Write the product: $Av = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 7 \\ -1 \end{bmatrix}$.
- Multiply the first row: $4(7) + (-3)(-1) = 31$.
- Multiply the second row: $2(7) + 5(-1) = 9$.

Answer: $Av = \begin{bmatrix} 31 \\ 9 \end{bmatrix}$

PRACTICE 6 problems

- 1** Let $M = \begin{bmatrix} 6 & 8 \\ -10 & 11 \end{bmatrix}$ and $v = \begin{bmatrix} 12 \\ -13 \end{bmatrix}$. Find Mv .

ANSWER _____

- 2** Let $N = \begin{bmatrix} 14 & -15 \\ 16 & 17 \end{bmatrix}$ and $w = \begin{bmatrix} -18 \\ 19 \end{bmatrix}$. Find Nw .

ANSWER _____

- 3** Let $P = \begin{bmatrix} 0 & -6 \\ 8 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 22 \\ 23 \end{bmatrix}$. Find Pu .

ANSWER _____

- 4** Let $T = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 10 \end{bmatrix}$ and $r = \begin{bmatrix} 11 \\ -12 \\ 13 \end{bmatrix}$. Find Tr .

ANSWER _____

- 5** Let $Q = \begin{bmatrix} -14 & 15 \\ 16 & -17 \end{bmatrix}$ and $s = \begin{bmatrix} 18 \\ 20 \end{bmatrix}$. Find Qs .

ANSWER _____

- 6** Let $R = \begin{bmatrix} 21 & -22 \\ 23 & 24 \end{bmatrix}$ and $h = \begin{bmatrix} -25 \\ 26 \end{bmatrix}$. Find Rh .

ANSWER _____

APPLY IT Show your work

- 1** A student uses a game-design app that transforms a character's position vector p by Gp , where $G = \begin{bmatrix} -6 & 8 \\ 10 & -11 \end{bmatrix}$. The character starts at $p = \begin{bmatrix} 12 \\ 14 \end{bmatrix}$. Find the transformed position vector.

WORK SPACE

ANSWER _____

- 2** A robotics team uses a matrix T to map a joystick input vector j to three motor commands. If $T = \begin{bmatrix} -6 & 0 & 8 \\ 0 & 10 & 0 \\ 12 & 0 & -14 \end{bmatrix}$ and $j = \begin{bmatrix} 15 \\ -16 \\ 17 \end{bmatrix}$, find Tj .

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let $C = \begin{bmatrix} -6 & 8 & 0 \\ 10 & -11 & 12 \\ 0 & 14 & -15 \end{bmatrix}$ and $q = \begin{bmatrix} 16 \\ -17 \\ 18 \end{bmatrix}$. Find Cq . Then explain in one sentence why Cq has three entries.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Let $M = \begin{bmatrix} 6 & 8 \\ -10 & 11 \end{bmatrix}$ and $v = \begin{bmatrix} 12 \\ -13 \end{bmatrix}$. Find Mv .	$Mv = \begin{bmatrix} -32 \\ -263 \end{bmatrix}$
2	Let $N = \begin{bmatrix} 14 & -15 \\ 16 & 17 \end{bmatrix}$ and $w = \begin{bmatrix} -18 \\ 19 \end{bmatrix}$. Find Nw .	$Nw = \begin{bmatrix} -537 \\ 35 \end{bmatrix}$
3	Let $P = \begin{bmatrix} 0 & -6 \\ 8 & 0 \end{bmatrix}$ and $u = \begin{bmatrix} 22 \\ 23 \end{bmatrix}$. Find Pu .	$Pu = \begin{bmatrix} -138 \\ 176 \end{bmatrix}$
4	Let $T = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 10 \end{bmatrix}$ and $r = \begin{bmatrix} 11 \\ -12 \\ 13 \end{bmatrix}$. Find Tr .	$Tr = \begin{bmatrix} 66 \\ 96 \\ 130 \end{bmatrix}$
5	Let $Q = \begin{bmatrix} -14 & 15 \\ 16 & -17 \end{bmatrix}$ and $s = \begin{bmatrix} 18 \\ 20 \end{bmatrix}$. Find Qs .	$Qs = \begin{bmatrix} 48 \\ -52 \end{bmatrix}$
6	Let $R = \begin{bmatrix} 21 & -22 \\ 23 & 24 \end{bmatrix}$ and $h = \begin{bmatrix} -25 \\ 26 \end{bmatrix}$. Find Rh .	$Rh = \begin{bmatrix} -1097 \\ 49 \end{bmatrix}$

PART 2 • APPLY IT

1. $Gp = \begin{bmatrix} 40 \\ -34 \end{bmatrix}$ Multiply each row of G by the position vector: $-6(12) + 8(14) = 40$ and $10(12) + (-11)(14) = -34$.
2. $Tj = \begin{bmatrix} 46 \\ -160 \\ -58 \end{bmatrix}$ Multiply each row by j : $-6(15) + 8(17) = 46$, $10(-16) = -160$, and $12(15) + (-14)(17) = -58$.

CHALLENGE

$Cq = \begin{bmatrix} -232 \\ 563 \\ -508 \end{bmatrix}$. It has three entries because each of the matrix's three rows produces one dot product.

Accept correctly formatted column vectors and equivalent expressions that simplify to the listed entries. In the challenge explanation, accept any accurate statement that connects one output entry to each matrix row.