

Matrix Area Moves

Use determinants to measure how transformations change area.

HIGH SCHOOL • HSN-VM.C.12

CCSS.Math.Content.HSN-VM.C.12 "(+) Work with 2 $\times$ 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area."

NAME _____

DATE _____

AREA SCALE FACTOR

A 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ changes areas by a scale factor of $|ad - bc|$. A negative determinant reverses orientation, but area is still nonnegative. If the determinant is 0, a two-dimensional region is flattened and its image has area 0.

WORKED EXAMPLE

A transformation has matrix $T = \begin{bmatrix} 11 & 13 \\ 17 & 19 \end{bmatrix}$. Find its area scale factor.

- $\det(T) = 11(19) - 13(17)$.
- $\det(T) = 209 - 221 = -12$.
- The area scale factor is $|-12| = 12$.

Answer: Areas are multiplied by 12.

PRACTICE 6 problems

- 1** For $T = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$, find $\det(T)$ and the area scale factor.

ANSWER _____

- 2** For $T = \begin{bmatrix} 6 & 10 \\ 9 & 15 \end{bmatrix}$, find $\det(T)$. What happens to the area of any region under T ?

ANSWER _____

- 3** For $T = \begin{bmatrix} 5 & 9 \\ 8 & 6 \end{bmatrix}$, find the determinant, the area scale factor, and the orientation change.

ANSWER _____

- 4** For $T = \begin{bmatrix} 0 & 14 \\ -5 & 0 \end{bmatrix}$, find the area scale factor. Does T preserve or reverse orientation?

ANSWER _____

- 5** A triangle has area 9 square units. It is transformed by $T = \begin{bmatrix} 8 & 0 \\ 0 & -5 \end{bmatrix}$. Find the area of the image triangle.

ANSWER _____

- 6** A rectangular photo region has area 4 square units. A graphics program applies $T = \begin{bmatrix} -8 & 0 \\ 0 & 6 \end{bmatrix}$. Find the area of the transformed region.

ANSWER _____

APPLY IT Show your work

1

A student design team has a digital poster shape with area 15 square inches. Their animation software applies the transformation with matrix $\begin{bmatrix} 4 & 1 \\ 2 & 6 \end{bmatrix}$. What is the area of the transformed shape?

WORK SPACE

ANSWER _____

2

A robotics club's camera map marks a safety zone with area 8 square units. The map is transformed by matrix $\begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix}$. What is the area of the transformed safety zone?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A matrix $T = \begin{bmatrix} x & 6 \\ 4 & 3 \end{bmatrix}$ transforms a region with area 10 square units into a region with area 180 square units. Find all values of x . Explain why there are two possible values.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For $T = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$, find $\det(T)$ and the area scale factor.	$\det(T) = 35$; area scale factor = 35.
2	For $T = \begin{bmatrix} 6 & 10 \\ 9 & 15 \end{bmatrix}$, find $\det(T)$. What happens to the area of any region under T ?	$\det(T) = 0$; every region has image area 0.
3	For $T = \begin{bmatrix} 5 & 9 \\ 8 & 6 \end{bmatrix}$, find the determinant, the area scale factor, and the orientation change.	$\det(T) = -42$; area scale factor = 42; orientation reverses.
4	For $T = \begin{bmatrix} 0 & 14 \\ -5 & 0 \end{bmatrix}$, find the area scale factor. Does T preserve or reverse orientation?	Area scale factor = 70; T preserves orientation.
5	A triangle has area 9 square units. It is transformed by $T = \begin{bmatrix} 8 & 0 \\ 0 & -5 \end{bmatrix}$. Find the area of the image triangle.	360 square units.
6	A rectangular photo region has area 4 square units. A graphics program applies $T = \begin{bmatrix} -8 & 0 \\ 0 & 6 \end{bmatrix}$. Find the area of the transformed region.	192 square units.

PART 2 • APPLY IT

- 330 square inches.** The determinant is $4(6) - 1(2) = 22$, so the area scale factor is 22. Multiply 15 by 22 to get 330 square inches.
- 208 square units.** The determinant is $(-3)(2) - 4(5) = -26$, so the area scale factor is 26. Multiply 8 by 26 to get 208 square units.

CHALLENGE

$x = 2$ or $x = 14$. The required area scale factor is $180/10 = 18$, so $|3x - 24| = 18$; solving gives both values.

Accept a correctly computed signed determinant when requested, but require its absolute value for any area scale factor or transformed area. For the challenge, accept $x = 2$ and $x = 14$ in either order, with reasoning based on $|3x - 24| = 18$.