LESSON MAKER PRO

Area Under Matrices

Use determinants to track how transformations change area.

HIGH SCHOOL · HSN-VM.C.12

CCSS.Math.Content.HSN-VM.C.12 "(+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area."

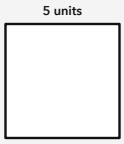
NAME _____ DATE _____

AREA SCALE FACTOR

For A = [[a, b], [c, d]], the area scale factor is |ad - bc|. Multiply the original area by this factor to find the image area.

WORKED EXAMPLE

Matrix A = [[11, 12], [13, 14]] maps the square in the figure. Find the area of the image.



1. det(A) = (11)(14) - (12)(13) = -2.
2. The area scale factor is |-2| = 2.
3. The original area is 5 × 5 = 25 square units.
4. Image area = 25 × 2 = 50 square units.

Answer: 50 square units

PRACTICE 4 problems

1 For A = [[3, 1], [1, 3]], find the area scale factor.

ANSWER _____

2 A triangle with area 6 square units is mapped by A = [[1, 3], [2, 7]]. Find the image area.

ANSWER _____

3 A region is mapped by A = [[6, -1], [1, 3]]. Its image has area 76 square units. Find the original area.

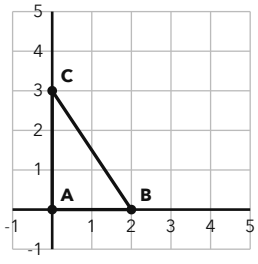
ANSWER _____

4 A region with area 9 square units is mapped by A = [[8, 2], [4, 1]]. Find the image area and state what the determinant tells you.

ANSWER _____

APPLY IT Show your work

1 A sticker design is the solid triangle on the grid. A printer maps it using A = [[2, 1], [1, 3]]. What is the area of the printed design?



ANSWER _____

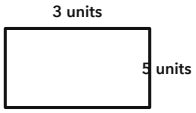
2 A concert banner has area 12 square feet. A digital editor maps it using A = [[3, 1], [1, 1]]. What is the area of the edited banner?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The rectangle in the figure is mapped by A = [[1, 2], [2, 4]]. Explain why its image has area 0 square units, even though the original rectangle has positive area.



WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For A = [[3, 1], [1, 3]], find the area scale factor.	**8**
2	A triangle with area 6 square units is mapped by A = [[1, 3], [2, 7]]. Find the image area.	**6 square units**
3	A region is mapped by A = [[6, -1], [1, 3]]. Its image has area 76 square units. Find the original area.	**4 square units**
4	A region with area 9 square units is mapped by A = [[8, 2], [4, 1]]. Find the image area and state what the determinant tells you.	**0 square units; the transformation collapses the region into a line segment.**

PART 2 · APPLY IT

1. **15 square units** The triangle has area (1/2)(2)(3) = 3 square units. Since |det(A)| = |6 - 1| = 5, the image area is 3 × 5 = 15 square units.
2. **24 square feet** The area scale factor is |(3)(1) - (1)(1)| = 2. Multiply 12 by 2.

CHALLENGE

det(A) = (1)(4) - (2)(2) = 0, so the area scale factor is 0. The transformation maps the rectangle into a line segment, so its image has area 0 square units.

Accept equivalent determinant calculations and correct units. For the challenge, accept any explanation that connects determinant 0 to a collapsed, zero-area image.