LESSON MAKER PRO

Matrix Order Matters

Compare products, group factors, and distribute matrices.

HIGH SCHOOL · HSN-VM.C.9

CCSS.Math.Content.HSN-VM.C.9 "(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties."

NAME ______________________ DATE ______________

THE BIG IDEA

For square matrices A and B, calculate AB by multiplying rows of A by columns of B. Usually AB does not equal BA, but matrices do satisfy (AB)C = A(BC) and A(B + C) = AB + AC.

WORKED EXAMPLE

Let A = [[6, 7], [8, 9]] and B = [[10, 11], [12, 13]]. Find AB and BA.

1. AB = [[6(10) + 7(12), 6(11) + 7(13)], [8(10) + 9(12), 8(11) + 9(13)]]
2. AB = [[144, 157], [188, 205]]
3. BA = [[10(6) + 11(8), 10(7) + 11(9)], [12(6) + 13(8), 12(7) + 13(9)]]
4. BA = [[148, 169], [176, 201]]

Answer: AB = [[144, 157], [188, 205]]; BA = [[148, 169], [176, 201]], so AB does not equal BA.

PRACTICE

6 problems

1 Let A = [[1, 2], [3, 0]] and B = [[4, 1], [2, 5]]. Find AB and BA. Do these matrices commute?

ANSWER ______________________

2 Let C = [[0, 1], [2, 1]] and D = [[3, 2], [1, 0]]. Find CD and DC.

ANSWER ______________________

3 Let E = [[1, 0], [0, 2]], F = [[2, 1], [0, 1]], and G = [[1, 1], [1, 0]]. Find (EF)G and E(FG).

ANSWER ______________________

4 Let H = [[2, 1], [1, 1]], J = [[1, 2], [0, 1]], and K = [[2, 0], [1, 3]]. Find H(J + K) and HJ + HK.

ANSWER ______________________

5 Let M = [[1, 3], [2, 1]] and N = [[0, 2], [1, 1]]. Find MN - NM.

ANSWER ______________________

6 Let P = [[3, 1], [0, 2]], Q = [[2, 1], [1, 3]], and R = [[1, 0], [2, 1]]. Find P(Q - R) and PQ - PR.

ANSWER ______________________

APPLY IT
Show your work

1 A music app represents bass and treble settings with a column vector. Its Boost setting uses X = [[1, 1], [0, 1]], and its Volume setting uses Y = [[2, 0], [0, 1]]. For any input vector, find the combined matrix for applying Boost then Volume. Then find the combined matrix for applying Volume then Boost. Are the results the same?

WORK SPACE

ANSWER ______________________

2 A game design club applies three transformation matrices to a character's position vector. The character receives C first, then B, then A, where A = [[1, 1], [0, 1]], B = [[1, 0], [1, 1]], and C = [[2, 0], [0, 1]]. One coder calculates A(BC), while another calculates (AB)C. Find both products.

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

Let A = [[1, x], [0, 1]] and B = [[1, 0], [y, 1]], where x and y are real numbers. Find all values of x and y for which AB = BA. Explain your reasoning.

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ANSWER KEY **Matrix Order Matters** HIGH SCHOOL · HSN-VM.C.9

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let A = [[1, 2], [3, 0]] and B = [[4, 1], [2, 5]]. Find AB and BA. Do these matrices commute?	**AB = [[8, 11], [12, 3]]; BA = [[7, 8], [17, 4]]. No, they do not commute.**
2	Let C = [[0, 1], [2, 1]] and D = [[3, 2], [1, 0]]. Find CD and DC.	**CD = [[1, 0], [7, 4]]; DC = [[4, 5], [0, 1]].**
3	Let E = [[1, 0], [0, 2]], F = [[2, 1], [0, 1]], and G = [[1, 1], [1, 0]]. Find (EF)G and E(FG).	**(EF)G = [[3, 2], [2, 0]]; E(FG) = [[3, 2], [2, 0]].**
4	Let H = [[2, 1], [1, 1]], J = [[1, 2], [0, 1]], and K = [[2, 0], [1, 3]]. Find H(J + K) and HJ + HK.	**H(J + K) = [[7, 8], [4, 6]]; HJ + HK = [[7, 8], [4, 6]].**
5	Let M = [[1, 3], [2, 1]] and N = [[0, 2], [1, 1]]. Find MN - NM.	**MN - NM = [[-1, 3], [-2, 1]].**
6	Let P = [[3, 1], [0, 2]], Q = [[2, 1], [1, 3]], and R = [[1, 0], [2, 1]]. Find P(Q - R) and PQ - PR.	**P(Q - R) = [[2, 5], [-2, 4]]; PQ - PR = [[2, 5], [-2, 4]].**

PART 2 · APPLY IT

1. **Boost then Volume: YX = [[2, 2], [0, 1]]. Volume then Boost: XY = [[2, 1], [0, 1]]. The results are not the same.** With column vectors, the setting applied last is written on the left. Compute YX for Boost then Volume and XY for Volume then Boost.
2. **A(BC) = [[4, 1], [2, 1]]; (AB)C = [[4, 1], [2, 1]].** First find BC and AB, then multiply by the remaining matrix. Both groupings produce the same matrix because multiplication is associative.

CHALLENGE

AB = BA when xy = 0. Therefore, x = 0 or y = 0, including the case where both are 0.

Accept correctly formatted matrices and any valid row-by-column work. For the challenge, accept xy = 0 or the equivalent statement x = 0 or y = 0 for real x and y.