

Model Check

Use probability to test a claim.

HIGH SCHOOL • HSS-IC.A.2

CCSS.Math.Content.HSS-IC.A.2 "Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5..."

NAME _____

DATE _____

WHEN TO QUESTION A MODEL

Find the probability of the result, or a result at least that unusual, if the model is true. A result below 0.05 is unusual enough to question the model, but it does not prove the model is false.

WORKED EXAMPLE

A machine model says an item works with probability 0.6. In 10 tests, exactly 8 items work. Should you question the model?

1. $P(\text{exactly 8 work}) = C(10,8) (0.6)^8 (0.4)^2$.
2. $P = 45(0.6)^8 (0.4)^2 = 0.120932352$.
3. 120932352 is greater than 0.05.

Answer: No. This result has probability about 0.121, so it is not unusually rare.

PRACTICE 5 problems

- 1** A fair coin model predicts heads with probability $1/2$. You get 7 heads in a row. Find the probability. Question the model?

ANSWER _____

- 2** A model says a device succeeds with probability 0.7. It fails on 4 attempts in a row. Find the probability. Question the model?

ANSWER _____

- 3** A model says a player makes a free throw with probability 0.2. The player makes 6 in a row. Find the probability. Question the model?

ANSWER _____

- 4** A bag model says red is drawn with probability 0.4, with replacement. In 4 draws, exactly 3 are red. Find the probability. Question the model?

ANSWER _____

- 5** An app model says an update crashes with probability 0.1. Nine updates have no crashes. Find the probability. Question the model?

ANSWER _____

APPLY IT Show your work

- 1** A music app says each song in a shuffle has probability 0.25 of being explicit. A listener gets 5 explicit songs in a row. Should the listener question the shuffle model?

WORK SPACE

ANSWER _____

- 2** An esports player claims to win each round with probability $\frac{3}{4}$. In 7 rounds, the player loses every round. Should you question the claim?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A quiz app model says a random guess is correct with probability $\frac{1}{2}$. In 11 questions, a student gets 9 or more correct. Find the probability of 9, 10, or 11 correct answers. Should you question the model? Explain.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A fair coin model predicts heads with probability $1/2$. You get 7 heads in a row. Find the probability. Question the model?	$1/128$. Yes, because $1/128$ is less than 0.05.
2	A model says a device succeeds with probability 0.7. It fails on 4 attempts in a row. Find the probability. Question the model?	0.0081. Yes, because 0.0081 is less than 0.05.
3	A model says a player makes a free throw with probability 0.2. The player makes 6 in a row. Find the probability. Question the model?	0.000064. Yes, because 0.000064 is less than 0.05.
4	A bag model says red is drawn with probability 0.4, with replacement. In 4 draws, exactly 3 are red. Find the probability. Question the model?	0.1536. No, because 0.1536 is greater than 0.05.
5	An app model says an update crashes with probability 0.1. Nine updates have no crashes. Find the probability. Question the model?	0.387420489. No, because 0.387420489 is greater than 0.05.

PART 2 • APPLY IT

- Yes. The probability is $(0.25)^5 = 1/1024$, which is less than 0.05.** Multiply 0.25 by itself 5 times. The result is an unusually rare streak if the model is correct.
- Yes. The probability is $(1/4)^7 = 1/16384$, which is less than 0.05.** The probability of a loss is $1/4$. Multiply $1/4$ by itself 7 times and compare with 0.05.

CHALLENGE

$P = [C(11,9) + C(11,10) + C(11,11)]/2^{11} = (55 + 11 + 1)/2048 = 67/2048$, about 0.0327. Yes, because getting 9 or more correct has probability less than 0.05.

Accept exact fractions or accurate decimals. Accept reasoning that identifies the result as unusual under the stated 0.05 guideline.