

Chance to Score

Build distributions and find expected values.

HIGH SCHOOL • HSS-MD.A.3

CCSS.Math.Content.HSS-MD.A.3 "(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example..."

NAME _____

DATE _____

MAP THE OUTCOMES

List every value the random variable can take, then give the probability of each value. Multiply each value by its probability and add the products to find the expected value.

WORKED EXAMPLE

A fair 11-section spinner has labels 1 through 11. Let $X=7$ if it lands on 11 and $X=0$ otherwise. Find the distribution and $E(X)$.

1. Possible values: 0 and 7.
2. $P(X=0)=10/11$ and $P(X=7)=1/11$.
3. $E(X)=0(10/11)+7(1/11)=7/11$.

**Answer: Distribution: 0: 10/11, 7: 1/11.
 $E(X)=7/11$.**

PRACTICE 5 problems

- 1** Toss a fair coin twice. Let X be the number of heads. Find the distribution and $E(X)$.

ANSWER _____

- 2** Roll a fair 6-sided die. Let $X=1$ for an odd roll and $X=4$ for an even roll. Find the distribution and $E(X)$.

ANSWER _____

- 3** You guess on 3 true-false questions. Let X be the number correct. Find the distribution and $E(X)$.

ANSWER _____

- 4** Choose one card at random from cards numbered 1, 2, 3, and 4. Let X be the card number. Find the distribution and $E(X)$.

ANSWER _____

- 5** A fair spinner has three equal sections worth 0, 2, and 6 points. Let X be your score. Find the distribution and $E(X)$.

ANSWER _____

APPLY IT Show your work

- 1** At a school-store event, you pick one of five equally likely mystery coupons. The coupon values are \$0, \$0, \$1, \$3, and \$6. Let X be the coupon value. Give the distribution and expected value.

WORK SPACE

ANSWER _____

- 2** You guess on a 4-question quiz with 3 choices per question. You earn 2 points for each correct answer. Let S be your quiz score. Give the distribution of S and $E(S)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Roll two fair 4-sided dice labeled 0, 1, 2, and 3. Let X be the product of the two results. Find the distribution and $E(X)$. Explain why your probabilities add to 1.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Toss a fair coin twice. Let X be the number of heads. Find the distribution and $E(X)$.	0: $1/4$, 1: $1/2$, 2: $1/4$. $E(X)=1$.
2	Roll a fair 6-sided die. Let $X=1$ for an odd roll and $X=4$ for an even roll. Find the distribution and $E(X)$.	1: $1/2$, 4: $1/2$. $E(X)=5/2$.
3	You guess on 3 true-false questions. Let X be the number correct. Find the distribution and $E(X)$.	0: $1/8$, 1: $3/8$, 2: $3/8$, 3: $1/8$. $E(X)=3/2$.
4	Choose one card at random from cards numbered 1, 2, 3, and 4. Let X be the card number. Find the distribution and $E(X)$.	1: $1/4$, 2: $1/4$, 3: $1/4$, 4: $1/4$. $E(X)=5/2$.
5	A fair spinner has three equal sections worth 0, 2, and 6 points. Let X be your score. Find the distribution and $E(X)$.	0: $1/3$, 2: $1/3$, 6: $1/3$. $E(X)=8/3$.

PART 2 • APPLY IT

- 0: $2/5$, 1: $1/5$, 3: $1/5$, 6: $1/5$. $E(X)=\$2$.** Combine the two \$0 coupons into $P(X=0)=2/5$. Add each coupon value times its probability.
- 0: $16/81$, 2: $32/81$, 4: $8/27$, 6: $8/81$, 8: $1/81$. $E(S)=8/3$ points.** The number correct follows 4 independent guesses with $P(\text{correct})=1/3$. Multiply each possible number correct by 2 to convert the distribution to scores.

CHALLENGE

0: $7/16$, 1: $1/16$, 2: $1/8$, 3: $1/8$, 4: $1/16$, 6: $1/8$, 9: $1/16$. $E(X)=9/4$. The 16 ordered pairs are equally likely, and the outcome counts total 16.

Accept equivalent fractions and distributions shown as tables, lists, or equations. For the quiz item, $24/81$ is equivalent to $8/27$.