

Expected Value Lab

Build data-based distributions and make predictions.

HIGH SCHOOL • HSS-MD.A.4

CCSS.Math.Content.HSS-MD.A.4 "(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example..."

NAME _____

DATE _____

FROM DATA TO PREDICTION

An empirical probability is an observed frequency divided by the total number of observations. Find $E(x)$ by adding each value times its probability. Multiply $E(x)$ by a future sample size to predict a total.

WORKED EXAMPLE

A survey records the number of podcasts heard last week: $x = 4, 7, 9$ with frequencies 1, 2, 5. Find the distribution and $E(x)$.

1. Total observations = $1 + 2 + 5 = 8$.
2. $P(4)=1/8$, $P(7)=2/8$, $P(9)=5/8$.
3. $E(x)=4(1/8)+7(2/8)+9(5/8)=63/8$.

**Answer: Distribution: $1/8, 1/4, 5/8$.
 $E(x)=63/8$.**

PRACTICE 5 problems

- 1** A sample gives $x = 0, 1, 2$ with frequencies 5, 7, 8. Find the distribution and $E(x)$.

ANSWER _____

- 2** A sample gives $x = 2, 3$ with frequencies 10, 14. Find $E(x)$.

ANSWER _____

- 3** A sample gives $x = 0, 1, 2, 3$ with frequencies 12, 10, 6, 2. Find $E(x)$.

ANSWER _____

- 4** A sample gives $x = 1, 2, 3$ with frequencies 16, 20, 4. Find $E(x)$, then predict the total for 60 new observations.

ANSWER _____

- 5** In a survey, the number of siblings was 0 for 14 students, 1 for 18 students, and 2 for 8 students. Find $E(x)$.

ANSWER _____

APPLY IT Show your work

- 1** A student council surveys 30 students about school events attended last month. The results are 0 events for 5 students, 1 event for 10 students, 2 events for 9 students, and 3 events for 6 students. Find the expected number of events per student. Predict the total events for 75 students.

WORK SPACE

ANSWER _____

- 2** A game club surveys 40 students about games borrowed from the school library this month. The results are 0 games for 12 students, 1 game for 16 students, 2 games for 8 students, and 3 games for 4 students. Predict the number of games 200 students would borrow.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two classes survey students about volunteer events attended this semester. Class A has 20 students: 0 events for 8, 1 event for 10, and 2 events for 2. Class B has 30 students: 0 events for 6, 1 event for 12, and 2 events for 12. Combine the data. Give the pooled distribution, $E(x)$, and the predicted total for 120 students. Explain why you should not simply average the two class expected values.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A sample gives $x = 0, 1, 2$ with frequencies 5, 7, 8. Find the distribution and $E(x)$.	$P(0)=1/4, P(1)=7/20, P(2)=2/5;$ $E(x)=23/20.$
2	A sample gives $x = 2, 3$ with frequencies 10, 14. Find $E(x)$.	$E(x)=31/12.$
3	A sample gives $x = 0, 1, 2, 3$ with frequencies 12, 10, 6, 2. Find $E(x)$.	$E(x)=14/15.$
4	A sample gives $x = 1, 2, 3$ with frequencies 16, 20, 4. Find $E(x)$, then predict the total for 60 new observations.	$E(x)=17/10$; predicted total = 102.
5	In a survey, the number of siblings was 0 for 14 students, 1 for 18 students, and 2 for 8 students. Find $E(x)$.	$E(x)=17/20.$

PART 2 · APPLY IT

- $E(x)=23/15$ events; predicted total = 115 events.** Use frequencies over 30 to find $E(x)=(0 \cdot 5 + 1 \cdot 10 + 2 \cdot 9 + 3 \cdot 6)/30 = 23/15$. Then multiply $23/15$ by 75.
- 220 games.** $E(x)=(0 \cdot 12 + 1 \cdot 16 + 2 \cdot 8 + 3 \cdot 4)/40 = 11/10$ games per student. Multiply $11/10$ by 200.

CHALLENGE

$P(0)=7/25, P(1)=11/25, P(2)=7/25; E(x)=1$; predicted total = 120 events. The classes have different sample sizes, so their expected values must be weighted by 20 and 30, not averaged equally.

Accept equivalent fractions and terminating decimals for expected values and probabilities. For predictions, accept the exact expected total, even though an actual future total may differ.