

Choice by Chance

Use expected value to compare costs, risks, and rewards.

HIGH SCHOOL • HSS-MD.B.5b

CCSS.Math.Content.HSS-MD.B.5b "Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident."

NAME _____

DATE _____

COMPARE THE AVERAGES

To find an expected value, multiply each possible outcome by its probability, then add the products. To compare strategies, include every cost or gain that belongs to each strategy, then choose the larger expected gain or smaller expected cost.

WORKED EXAMPLE

A raffle ticket costs \$16. It has a $\frac{1}{8}$ chance to win \$64 and otherwise wins \$0. What is the expected net value of one ticket?

- Expected winnings = $(\frac{1}{8}) \times 64 + (\frac{7}{8}) \times 0$.
- Expected winnings = \$8.
- Expected net value = $\$8 - \$16 = -\$8$.

Answer: -\$8 per ticket

PRACTICE 6 problems

- 1** A service plan charges a \$9 fee. There is a $\frac{1}{3}$ chance that you will also pay a \$27 repair charge. Find the expected cost.

ANSWER _____

- 2** A game has a $\frac{1}{5}$ chance to win \$35 and otherwise wins nothing. The entry fee is \$11. Find the expected net value.

ANSWER _____

- 3** Strategy R always costs \$14. Strategy S has a $\frac{1}{2}$ chance to cost \$30 and otherwise has no cost. Which strategy has the lower expected cost?

ANSWER _____

- 4** Policy A has a \$21 premium and a $\frac{1}{7}$ chance of a \$49 deductible. Policy B has a \$26 premium and a $\frac{1}{7}$ chance of a \$7 deductible. Which policy has the lower expected cost?

ANSWER _____

- 5** A prize wheel pays \$24 with probability $\frac{1}{6}$, pays \$12 with probability $\frac{1}{4}$, and pays nothing for all other outcomes. Find the expected payout.

ANSWER _____

- 6** Option X has a $\frac{1}{10}$ chance to cost \$50, a $\frac{3}{10}$ chance to cost \$20, and otherwise has no cost. Option Y has a fixed cost of \$12. Which option has the lower expected cost?

ANSWER _____

APPLY IT Show your work

- 1** Sam is deciding whether to buy a \$24 screen protection plan for a tablet. With the plan, Sam still pays a \$6 deductible if the screen breaks. Without the plan, a screen repair costs \$96. The chance of a broken screen during the year is $\frac{1}{4}$. Which choice has the lower expected cost?

WORK SPACE

ANSWER _____

- 2** A student can buy a \$15 bike lock. With the lock, the chance that the bike is stolen during the year is $\frac{1}{20}$. Without a lock, the chance is $\frac{1}{5}$. Replacing a stolen bike costs \$150. Which strategy has the lower expected cost?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two auto insurance policies cover the same car. Policy A has a \$72 premium, a $\frac{1}{10}$ chance of a minor accident with a \$180 deductible, and a $\frac{1}{20}$ chance of a major accident with a \$480 deductible. Policy B has a \$108 premium, a $\frac{1}{10}$ chance of a minor accident with a \$60 deductible, and a $\frac{1}{20}$ chance of a major accident with a \$120 deductible. Which policy has the lower expected yearly cost? Show how both accident types affect your comparison.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A service plan charges a \$9 fee. There is a $\frac{1}{3}$ chance that you will also pay a \$27 repair charge. Find the expected cost.	\$18
2	A game has a $\frac{1}{5}$ chance to win \$35 and otherwise wins nothing. The entry fee is \$11. Find the expected net value.	-\$4
3	Strategy R always costs \$14. Strategy S has a $\frac{1}{2}$ chance to cost \$30 and otherwise has no cost. Which strategy has the lower expected cost?	Strategy R. Its expected cost is \$14, compared with \$15 for Strategy S.
4	Policy A has a \$21 premium and a $\frac{1}{7}$ chance of a \$49 deductible. Policy B has a \$26 premium and a $\frac{1}{7}$ chance of a \$7 deductible. Which policy has the lower expected cost?	Policy B. Policy A has an expected cost of \$28, and Policy B has an expected cost of \$27.
5	A prize wheel pays \$24 with probability $\frac{1}{6}$, pays \$12 with probability $\frac{1}{4}$, and pays nothing for all other outcomes. Find the expected payout.	\$7
6	Option X has a $\frac{1}{10}$ chance to cost \$50, a $\frac{3}{10}$ chance to cost \$20, and otherwise has no cost. Option Y has a fixed cost of \$12. Which option has the lower expected cost?	Option X. Its expected cost is \$11, compared with \$12 for Option Y.

PART 2 • APPLY IT

- Without the plan. With the plan, the expected cost is $\$51/2$, or \$25.50. Without the plan, the expected cost is \$24.**
With the plan, add the \$24 plan cost and $(\frac{1}{4}) \times \$6$. Without the plan, calculate $(\frac{1}{4}) \times \$96$.
- Buy the lock. With the lock, the expected cost is $\$45/2$, or \$22.50. Without a lock, the expected cost is \$30.** For the lock strategy, add \$15 and $(\frac{1}{20}) \times \$150$. For no lock, calculate $(\frac{1}{5}) \times \$150$.

CHALLENGE

Policy A has the lower expected yearly cost. Policy A: $\$72 + (\frac{1}{10} \times \$180) + (\frac{1}{20} \times \$480) = \$72 + \$18 + \$24 = \114 .
Policy B: $\$108 + (\frac{1}{10} \times \$60) + (\frac{1}{20} \times \$120) = \$108 + \$6 + \$6 = \120 . **Policy A is lower by \$6.**

Accept equivalent fractions, decimals, or money amounts with trailing zeros. For comparison problems, accept the correct strategy only when the expected values or a valid calculation are shown.