

Smart Odds Choices

Use expected value to compare risky options.

HIGH SCHOOL · HSS-MD.B.7

CCSS.Math.Content.HSS-MD.B.7 "(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)."

NAME _____

DATE _____

EXPECTED VALUE

Expected value predicts the average net result of a choice over many repeats. Multiply each outcome by its probability, use negative values for losses, and add the results.

WORKED EXAMPLE

A game has a $\frac{1}{8}$ chance to win \$24 and a $\frac{7}{8}$ chance to lose \$4. Find the expected value.

1. Write the outcomes as 24 and -4.
2. $EV = (\frac{1}{8})(24) + (\frac{7}{8})(-4)$.
3. $EV = 3 - 3 = 0$.

Answer: \$0 expected value

PRACTICE 5 problems

- 1** A game has a $\frac{2}{3}$ chance to win \$9 and otherwise loses \$3. Find its expected value.

ANSWER _____

- 2** A coupon has a $\frac{3}{5}$ chance to save \$20 and otherwise costs \$5. Find its expected value.

ANSWER _____

- 3** A contest has a $\frac{2}{5}$ chance to pay \$15 and otherwise charges \$5. Find its expected value.

ANSWER _____

- 4** Option A has a $\frac{2}{3}$ chance to gain \$12 and otherwise loses \$6. Option B has a $\frac{3}{5}$ chance to gain \$10 and otherwise loses \$5. Which option has the greater expected value?

ANSWER _____

- 5** A mystery box has a $\frac{3}{10}$ chance to contain a \$30 gift card and otherwise contains nothing. What is a fair price for the box?

ANSWER _____

APPLY IT Show your work

- 1** A robotics club can sell one of two hoodie designs. Design A has a $\frac{2}{5}$ chance to earn \$45 profit and otherwise loses \$15. Design B guarantees a \$6 profit. Which design should the club choose based on expected value?

WORK SPACE

ANSWER _____

- 2** A student musician can buy a promotion for \$12. The promotion has a $\frac{3}{5}$ chance to bring in \$25 in extra sales and otherwise brings in \$0. Should the musician buy it based on expected value?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A game developer can test a new feature for \$5. A serious bug has probability $\frac{2}{5}$. If a bug exists, the test catches it with probability $\frac{2}{3}$, and an early repair costs \$6. A missed bug costs \$30 after release. If no test is used, any bug costs \$30 after release. Which plan has the lower expected cost? Show the expected cost of each plan.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A game has a $\frac{2}{3}$ chance to win \$9 and otherwise loses \$3. Find its expected value.	\$5
2	A coupon has a $\frac{3}{5}$ chance to save \$20 and otherwise costs \$5. Find its expected value.	\$10
3	A contest has a $\frac{2}{5}$ chance to pay \$15 and otherwise charges \$5. Find its expected value.	\$3
4	Option A has a $\frac{2}{3}$ chance to gain \$12 and otherwise loses \$6. Option B has a $\frac{3}{5}$ chance to gain \$10 and otherwise loses \$5. Which option has the greater expected value?	Option A, with an expected value of \$6. Option B has an expected value of \$4.
5	A mystery box has a $\frac{3}{10}$ chance to contain a \$30 gift card and otherwise contains nothing. What is a fair price for the box?	\$9

PART 2 • APPLY IT

- Choose Design A. Its expected value is \$9, compared with \$6 for Design B.** Design A: $(\frac{2}{5})(45) + (\frac{3}{5})(-15) = 18 - 9 = 9$. Compare \$9 with the guaranteed \$6.
- Yes. The expected net value is \$3.** Expected sales are $(\frac{3}{5})(25) = 15$. Subtract the \$12 cost to get a net expected value of \$3.

CHALLENGE

Test the feature. Testing has expected cost $\$53/5$, or \$10.60: $5 + (\frac{2}{5})[(\frac{2}{3})(6) + (\frac{1}{3})(30)] = 53/5$. No test has expected cost $(\frac{2}{5})(30) = \$12$.

Accept equivalent fraction, decimal, or currency forms. For decision items, accept the stated choice only when it is supported by the correct expected values.