

Index Detective

Find every allowed input in a sequence.

HIGH SCHOOL • MA.912.AR.10.6

MA.912.AR.10.6 "Given a mathematical or real-world context, find the domain of a given sequence defined recursively or explicitly."

NAME _____

DATE _____

ALLOWED INDICES

The domain of a sequence is the set of integer index values n that its definition or context permits. Read the starting and ending conditions carefully, and remove any index that makes an expression undefined, such as a zero denominator. A real-world situation can also limit the indices.

WORKED EXAMPLE

Find the domain of $a_n = 11n + 13$ for integers $n \geq 14$.

1. n must be an integer that is at least 14.
2. The allowed indices begin at 14 and continue without end.
3. Write the domain as $\{14, 15, 16, \dots\}$.

Answer: $\{14, 15, 16, \dots\}$

PRACTICE 6 problems

- 1** Find the domain of $a_n = 4n - 1$ for $n = 1, 2, 3, \dots$

ANSWER _____

- 2** Find the domain of the recursive sequence $b_0 = 7$ and $b_n = b_{(n-1)} + 3$ for $n \geq 1$.

ANSWER _____

- 3** Find the domain of $c_n = (-1)^n / (n + 5)$ for $n = 0, 1, 2, \dots$

ANSWER _____

- 4** Find the domain of $d_n = 3 / (n - 6)$, where n is a positive integer and d_n is defined.

ANSWER _____

- 5** Find the domain of the recursive sequence $e_3 = 9$ and $e_n = e_{(n-1)} - 2$ for $n \geq 4$.

ANSWER _____

- 6** Find the domain of $f_n = (n - 1)!$ for $n = 1, 2, 3, \dots$

ANSWER _____

APPLY IT Show your work

- 1** In a 10-match esports league, Maya earns 18 points after match 1. Her point total follows $K_1 = 18$ and $K_n = K_{(n-1)} - 2$ for $2 \leq n \leq 10$. What is the domain of K_n ?

WORK SPACE

ANSWER _____

- 2** For a social media posting challenge, Jordan records followers at the start of month 0 and through month 8. The sequence is $F_0 = 240$ and $F_n = F_{(n-1)} + 15$ for $1 \leq n \leq 8$. What is the domain of F_n ?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For integer n , $q_n = (3n + 1)/(n - 7)$, where $-3 \leq n \leq 9$. Find the domain of q_n .

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the domain of $a_n = 4n - 1$ for $n = 1, 2, 3, \dots$	{1, 2, 3, ...}
2	Find the domain of the recursive sequence $b_0 = 7$ and $b_n = b_{(n-1)} + 3$ for $n \geq 1$.	{0, 1, 2, 3, ...}
3	Find the domain of $c_n = (-1)^n/(n + 5)$ for $n = 0, 1, 2, \dots$	{0, 1, 2, 3, ...}
4	Find the domain of $d_n = 3/(n - 6)$, where n is a positive integer and d_n is defined.	{1, 2, 3, 4, 5, 7, 8, 9, ...}
5	Find the domain of the recursive sequence $e_3 = 9$ and $e_n = e_{(n-1)} - 2$ for $n \geq 4$.	{3, 4, 5, 6, ...}
6	Find the domain of $f_n = (n - 1)!$ for $n = 1, 2, 3, \dots$	{1, 2, 3, ...}

PART 2 • APPLY IT

- {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}** The index n represents a match number. The league includes matches 1 through 10 only.
- {0, 1, 2, 3, 4, 5, 6, 7, 8}** The index n represents the month number. The challenge records months 0 through 8.

CHALLENGE

{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 8, 9}

Accept equivalent set-builder notation, such as $\{n \text{ in } \mathbb{Z} \mid n \geq -3 \text{ and } n \neq 7\}$. For the challenge, students must exclude 7 because it makes the denominator zero.