

# Parabola Boundaries

*Solve quadratic inequalities by finding where signs change.*

**HIGH SCHOOL • MA.912.AR.3.3**

**MA.912.AR.3.3** "Given a mathematical or real-world context, write and solve one-variable quadratic inequalities over the real number system. Represent solutions algebraically or graphically."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## TRACK THE SIGNS

Factor the quadratic when possible, then use its zeros to split the number line into intervals. Test the sign of the factors in each interval, and include a zero for  $\leq$  or  $\geq$ , but not for  $<$  or  $>$ .

## WORKED EXAMPLE

Solve  $x^2 - 13x + 30 \geq 0$ .

1. Factor:  $(x - 3)(x - 10) \geq 0$ .
2. The zeros 3 and 10 split the number line into three intervals.
3. The product is positive outside the zeros.
4. Include both zeros because the inequality is  $\geq 0$ .

**Answer:  $x \leq 3$  or  $x \geq 10$**

## PRACTICE 6 problems

**1** Solve  $x^2 + x - 20 < 0$ .

**ANSWER** \_\_\_\_\_

**2** Solve  $x^2 - x - 2 \geq 0$ .

**ANSWER** \_\_\_\_\_

**3** Solve  $-x^2 + 9x - 14 > 0$ .

**ANSWER** \_\_\_\_\_

**4** Solve  $x^2 + 6x + 5 \leq 0$ .

**ANSWER** \_\_\_\_\_

**5** Solve  $(x - 4)(x + 6) > 0$ .

**ANSWER** \_\_\_\_\_

**6** Solve  $x^2 - 16 \leq 0$ .

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** At a school game, a spirit launcher sends a T-shirt upward. Its height in meters is modeled by  $h(x) = -x^2 + 12x + 7$ , where  $x$  is time in seconds. For what times is the T-shirt at least 7 meters high?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A student club models its profit from an online art sale by  $P(x) = -x^2 + 16x - 55$ , where  $P$  is in hundreds of dollars and  $x$  is the number of weeks the sale runs. For what values of  $x$  is the profit positive?

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Solve  $x^2 + 4x + 4 > 0$ . Explain why exactly one value of  $x$  is excluded.

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## PART 1 · PRACTICE

| # | PROBLEM                       | ANSWER                    |
|---|-------------------------------|---------------------------|
| 1 | Solve $x^2 + x - 20 < 0$ .    | $-5 < x < 4$              |
| 2 | Solve $x^2 - x - 2 \geq 0$ .  | $x \leq -1$ or $x \geq 2$ |
| 3 | Solve $-x^2 + 9x - 14 > 0$ .  | $2 < x < 7$               |
| 4 | Solve $x^2 + 6x + 5 \leq 0$ . | $-5 \leq x \leq -1$       |
| 5 | Solve $(x - 4)(x + 6) > 0$ .  | $x < -6$ or $x > 4$       |
| 6 | Solve $x^2 - 16 \leq 0$ .     | $-4 \leq x \leq 4$        |

## PART 2 · APPLY IT

- $0 \leq x \leq 12$**  Write  $-x^2 + 12x + 7 \geq 7$ , then factor to get  $-x(x - 12) \geq 0$ . Use the time restriction  $x \geq 0$ .
- $5 < x < 11$**  Set  $-x^2 + 16x - 55 > 0$  and factor as  $-(x - 5)(x - 11) > 0$ . The profit is positive between the two zeros.

## CHALLENGE

**$x < -2$  or  $x > -2$ . Since  $x^2 + 4x + 4 = (x + 2)^2$ , the expression equals 0 only when  $x = -2$  and is positive everywhere else.**

Accept equivalent interval notation and correctly shaded number-line graphs. For the first word problem, students must use the context restriction  $x \geq 0$ .