LESSON MAKER PRO

Parabola Pathways

Solve, graph, and interpret quadratic models.

HIGH SCHOOL · MA.912.AR.3.8

MA.912.AR.3.8 "Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context."

NAME _____ DATE _____

GRAPH THE KEY FEATURES

For y = a(x - h)^2 + k, the vertex is (h, k), the axis of symmetry is x = h, and the sign of a tells whether the graph opens up or down. To solve a quadratic, set it equal to 0 and factor when possible. In a real situation, use only x-values and y-values that make sense in the context.

WORKED EXAMPLE

Graph y = (x - 3)^2 - 7 by finding its vertex, axis of symmetry, and two symmetric points.

1. The vertex is (3, -7), so the axis of symmetry is x = 3.
2. Substitute x = 2: y = (2 - 3)^2 - 7 = -6.
3. By symmetry, when x = 4, y = -6.
4. Plot (3, -7), (2, -6), and (4, -6), then sketch a parabola opening up.

Answer: Vertex: (3, -7). Axis of symmetry: x = 3. Symmetric points: (2, -6) and (4, -6). The parabola opens up.

PRACTICE 6 problems

1 Solve x^2 - 14x + 45 = 0.

ANSWER _____

2 For f(x) = 5(x + 8)^2 - 13, state the vertex, axis of symmetry, and direction of opening.

ANSWER _____

3 For y = (x + 9)^2 - 15, list the vertex and one pair of symmetric points that can be used to graph the function.

ANSWER _____

4 For f(x) = -8(x - 10)^2 + 17, state the maximum value, the axis of symmetry, and the range.

ANSWER _____

5 Solve x^2 + 13x + 40 = 0.

ANSWER _____

6 Use the vertex and x-intercepts to graph y = x^2 - 18x + 80. State those key features.

ANSWER _____

APPLY IT Show your work

1 A ball-tracking app models the height of a basketball by h(t) = -2t^2 + 8t + 10, where h is height in feet and t is time in seconds after the toss. Graph only the meaningful part of the model. What is the ball's maximum height, when does it reach the ground, and what are the realistic domain and range?

WORK SPACE

ANSWER _____

2 For a gaming tournament, a student club models its profit by P(x) = -3x^2 + 36x - 81, where P is profit in tens of dollars and x is the number of snack bundles sold. The club can prepare at most 12 bundles. Graph the model for the context. How many bundles give the greatest profit, what is that profit, and how many bundles must be sold to avoid a loss?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

An experimental drone is on the ground at t = 1 and t = 9. Its height is modeled by h(t) = a(t - 1)(t - 9), and its maximum height is 64 meters. Find a, write the quadratic model, and state the realistic domain and range. Explain how the maximum height determines a.

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ANSWER KEY # Parabola Pathways

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve x^2 - 14x + 45 = 0.	**x = 5 or x = 9**
2	For f(x) = 5(x + 8)^2 - 13, state the vertex, axis of symmetry, and direction of opening.	**Vertex: (-8, -13). Axis of symmetry: x = -8. The parabola opens up.**
3	For y = (x + 9)^2 - 15, list the vertex and one pair of symmetric points that can be used to graph the function.	**Vertex: (-9, -15). One symmetric pair is (-10, -14) and (-8, -14).**
4	For f(x) = -8(x - 10)^2 + 17, state the maximum value, the axis of symmetry, and the range.	**Maximum value: 17. Axis of symmetry: x = 10. Range: y <= 17.**
5	Solve x^2 + 13x + 40 = 0.	**x = -5 or x = -8**
6	Use the vertex and x-intercepts to graph y = x^2 - 18x + 80. State those key features.	**Vertex: (9, -1). x-intercepts: (8, 0) and (10, 0). Axis of symmetry: x = 9. The parabola opens up.**

PART 2 · APPLY IT

1. **The vertex is (2, 18), so the maximum height is 18 feet. The ball reaches the ground at t = 5 seconds. Realistic domain: 0 <= t <= 5. Realistic range: 0 <= h <= 18.** The vertex occurs at t = -8/(2(-2)) = 2, and h(2) = 18. Factoring h(t) = -2(t - 5)(t + 1) shows that t = 5 is the meaningful ground time.
2. **Graph on the whole-number domain 0 <= x <= 12. The vertex is (6, 27), so selling 6 bundles gives the greatest profit, $270. The zeros are x = 3 and x = 9, so the club avoids a loss when it sells 3 through 9 bundles, inclusive.** Rewrite the model as P(x) = -3(x - 3)(x - 9), so the break-even values are 3 and 9. The axis of symmetry is halfway between them at x = 6, where P(6) = 27 tens of dollars.

CHALLENGE

a = -4, so h(t) = -4(t - 1)(t - 9), or h(t) = -4t^2 + 40t - 36. The vertex is (5, 64), so the realistic domain is 1 <= t <= 9 and the realistic range is 0 <= h <= 64. At t = 5, h(5) = a(4)(-4) = -16a; setting -16a = 64 gives a = -4.

Accept equivalent factored, standard, or vertex forms when they are correct. For graphs, accept accurate sketches that show the listed vertex, intercepts or points, axis of symmetry, direction of opening, and stated contextual restrictions.