

Parabola Boundary Zones

Turn curved boundaries into inequalities.

HIGH SCHOOL • MA.912.AR.3.9

MA.912.AR.3.9 "Given a mathematical or real-world context, write two-variable quadratic inequalities to represent relationships between quantities from a graph or a written description."

NAME _____

DATE _____

READ THE SHADING

Start with the equation of the parabola that forms the boundary, often $y = a(x-h)^2 + k$. If the shaded region is above the parabola, use $>$ or $>=$. If it is below, use $<$ or $<=$. Use $>=$ or $<=$ for a solid boundary because the curve is included, and use $>$ or $<$ for a dashed boundary because the curve is not included.

WORKED EXAMPLE

A dashed parabola has equation $y = 11(x-13)^2 + 17$. The region above the parabola is shaded. Write the inequality.

1. The boundary equation is $y = 11(x-13)^2 + 17$.
2. Shading above the boundary means y is greater than the boundary expression.
3. The dashed curve is not included, so use $>$.

Answer: $y > 11(x-13)^2 + 17$

PRACTICE 6 problems

- 1** A solid parabola has equation $y = x^2$. The region on and below the parabola is shaded. Write the inequality.

ANSWER _____

- 2** A dashed parabola has equation $y = -x^2 + 4$. The region above the parabola is shaded. Write the inequality.

ANSWER _____

- 3** The boundary is the solid parabola $y = (x+2)^2 + 1$. The region above the parabola is shaded. Write the inequality.

ANSWER _____

- 4** The boundary is the dashed parabola $y = 3(x-1)^2 - 6$. The region below the parabola is shaded. Write the inequality.

ANSWER _____

- 5** A graph shows the solid boundary $y = -2x^2 + 5$. The shaded region is on and below the curve. Write the inequality.

ANSWER _____

- 6** A dashed parabola has equation $y = \frac{1}{4}(x+4)^2 - 3$. The region above the parabola is shaded. Write the inequality.

ANSWER _____

APPLY IT Show your work

- 1** A student council designs a curved top for a concert-photo backdrop. On a coordinate grid, the top edge is $y = -x^2 + 8x + 2$. The printed backdrop includes the edge and every point below it. Write an inequality for points (x,y) on the backdrop.

WORK SPACE

ANSWER _____

- 2** At a skate park, the cross-section of a bowl's floor is $y = \frac{1}{2}(x-6)^2 - 7$. The riding space is on or above the floor, including the floor. Write an inequality for points (x,y) in the riding space.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A graph has a solid upward-opening parabola as its boundary. The parabola has vertex $(3,-4)$ and passes through $(1,4)$. The shaded region is on and below the parabola. Write the inequality.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A solid parabola has equation $y = x^2$. The region on and below the parabola is shaded. Write the inequality.	$y \leq x^2$
2	A dashed parabola has equation $y = -x^2 + 4$. The region above the parabola is shaded. Write the inequality.	$y > -x^2 + 4$
3	The boundary is the solid parabola $y = (x+2)^2 + 1$. The region above the parabola is shaded. Write the inequality.	$y \geq (x+2)^2 + 1$
4	The boundary is the dashed parabola $y = 3(x-1)^2 - 6$. The region below the parabola is shaded. Write the inequality.	$y < 3(x-1)^2 - 6$
5	A graph shows the solid boundary $y = -2x^2 + 5$. The shaded region is on and below the curve. Write the inequality.	$y \leq -2x^2 + 5$
6	A dashed parabola has equation $y = \frac{1}{4}(x+4)^2 - 3$. The region above the parabola is shaded. Write the inequality.	$y > \frac{1}{4}(x+4)^2 - 3$

PART 2 • APPLY IT

1. $y \leq -x^2 + 8x + 2$ The backdrop includes its curved edge, so the inequality includes equality. Points on the backdrop are below the top edge, so use $\leq$.
2. $y \geq \frac{1}{2}(x-6)^2 - 7$ The riding space is above the floor curve. Because the floor is included, use $\geq$.

CHALLENGE

$$y \leq 2(x-3)^2 - 4$$

Accept equivalent quadratic forms, including expanded forms, as long as the boundary equation and inequality symbol are correct. For the challenge, $y \leq 2x^2 - 12x + 14$ is equivalent.