

Exponential Pathways

Model, solve, graph, and interpret exponential change.

HIGH SCHOOL • MA.912.AR.5.7

MA.912.AR.5.7 "Solve and graph mathematical and real-world problems that are modeled with exponential functions. Interpret key features and determine constraints in terms of the context."

NAME _____

DATE _____

KEY FEATURES

For $y = a(b^x)$, a is the starting amount, b is the factor of change, and the y -intercept is $(0, a)$. If $b > 1$, the function grows. If $0 < b < 1$, it decays. The horizontal asymptote is often $y = 0$, but a real situation may restrict the input to values such as whole days or weeks.

WORKED EXAMPLE

Solve $11^x = 1331$.

1. Recognize that $1331 = 11^3$.
2. Rewrite the equation as $11^x = 11^3$.
3. Set the exponents equal: $x = 3$.

Answer: $x = 3$

PRACTICE 6 problems

- 1** For $f(x) = 18(1/2)^x$, state whether the function shows growth or decay. Then give the y -intercept and horizontal asymptote.

ANSWER _____

- 2** Graph $y = 5(4^x)$ by plotting the points for $x = -1, 0$, and 1 . State the horizontal asymptote.

ANSWER _____

- 3** Solve $6(2^x) = 96$.

ANSWER _____

- 4** Solve $16(1/2)^x = 1$.

ANSWER _____

- 5** Graph $y = 9(5^x)$ by plotting the points for $x = -2, 0$, and 2 . State the horizontal asymptote.

ANSWER _____

- 6** An exponential graph has y -intercept $(0, 14)$ and passes through $(1, 56)$. Write its equation in the form $y = a(b^x)$.

ANSWER _____

APPLY IT Show your work

- 1** A school club begins a fundraiser with \$25. The total amount doubles at the end of each week. Write a function $F(w)$ for the total after w weeks. Graph the model using $w = 0, 1, 2, 4$, and 5 . At what week will the total reach \$800? State a reasonable domain for w .

WORK SPACE

ANSWER _____

- 2** A video gets 640 new shares on day 0. The number of new shares each day is half the number from the previous day. Write a function $S(d)$ for the number of new shares after d days. Graph the model using $d = 0, 1, 2$, and 5 . How many new shares are predicted on day 5? State a reasonable domain for d .

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two online-post engagement models are $A(x) = 4(2^x)$ and $B(x) = 128(1/2)^x$, where x is time in hours after posting. At what time do the models have the same engagement score? Find that score and explain your exponent reasoning. State the time constraint.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = 18(1/2)^x$, state whether the function shows growth or decay. Then give the y-intercept and horizontal asymptote.	Decay; y-intercept: (0, 18); horizontal asymptote: $y = 0$
2	Graph $y = 5(4^x)$ by plotting the points for $x = -1, 0$, and 1 . State the horizontal asymptote.	Points: $(-1, 5/4)$, $(0, 5)$, $(1, 20)$; horizontal asymptote: $y = 0$
3	Solve $6(2^x) = 96$.	$x = 4$
4	Solve $16(1/2)^x = 1$.	$x = 4$
5	Graph $y = 9(5^x)$ by plotting the points for $x = -2, 0$, and 2 . State the horizontal asymptote.	Points: $(-2, 9/25)$, $(0, 9)$, $(2, 225)$; horizontal asymptote: $y = 0$
6	An exponential graph has y-intercept $(0, 14)$ and passes through $(1, 56)$. Write its equation in the form $y = a(b^x)$.	$y = 14(4^x)$

PART 2 · APPLY IT

- $F(w) = 25(2^w)$. Graph points: $(0, 25)$, $(1, 50)$, $(2, 100)$, $(4, 400)$, $(5, 800)$. The total reaches \$800 at $w = 5$. A reasonable domain is $w = 0, 1, 2, \dots$. Set $25(2^w)$ equal to 800 and divide by 25 to get $2^w = 32$. Since $32 = 2^5$, $w = 5$.**
- $S(d) = 640(1/2)^d$. Graph points: $(0, 640)$, $(1, 320)$, $(2, 160)$, $(5, 20)$. The prediction for day 5 is 20 new shares. A reasonable domain is $d = 0, 1, 2, \dots$. Substitute $d = 5$ into $S(d) = 640(1/2)^d$. Since 640 divided by 32 is 20, $S(5) = 20$.**

CHALLENGE

Set $4(2^x) = 128(1/2)^x$. Rewrite as $2^{(x+2)} = 2^{(7-x)}$, so $x+2 = 7-x$ and $x = 5/2$. The common score is $16\sqrt{2}$. The time constraint is $x \geq 0$.

Accept equivalent exponential forms and equivalent point notation. For contextual domains, accept nonnegative whole-number inputs when the situation is measured by complete days or weeks.