

Factor to Finish

Solve polynomial equations with real and complex roots.

HIGH SCHOOL • MA.912.AR.6.1

MA.912.AR.6.1 "Given a mathematical or real-world context, when suitable factorization is possible, solve one-variable polynomial equations of degree 3 or higher over the real and complex number systems."

NAME _____

DATE _____

FACTOR, THEN SOLVE

When a polynomial is written as factors and equals 0, set each factor equal to 0. For a real number a greater than 0, $x^2 + a^2 = 0$ gives $x = ai$ and $x = -ai$. If $a = 0$, $x^2 = 0$ has the real zero $x = 0$.

WORKED EXAMPLE

Solve $(x-7)(x+5)(x^2+121)=0$.

1. Set each factor equal to 0.
2. $x-7=0$ gives $x=7$, and $x+5=0$ gives $x=-5$.
3. $x^2+121=0$ gives $x^2=-121$, so $x=11i$ or $x=-11i$.

Answer: $x=7$, $x=-5$, $x=11i$, $x=-11i$

PRACTICE 6 problems

1 Solve $x^3 - x^2 - 12x = 0$.

ANSWER _____

2 Solve $x^3 + 6x^2 + 9x = 0$.

ANSWER _____

3 Solve $x^4 - 36x^2 = 0$.

ANSWER _____

4 Solve $x^4 + 15x^2 - 16 = 0$.

ANSWER _____

5 Solve $x^3 - 8 = 0$.

ANSWER _____

6 Solve $x^4 - 10x^2 + 9 = 0$.

ANSWER _____

APPLY IT Show your work

- 1** A maker club is printing a rectangular phone stand package. Its dimensions are x cm, $(x+2)$ cm, and $(x+4)$ cm, and its volume is 105 cubic centimeters. Find the positive value of x .

WORK SPACE

ANSWER _____

- 2** An esports club models the net earnings from a tournament after x weeks by $E(x)=x^4-6x^3+17x^2-54x+72$. At which nonnegative whole-number weeks does the model predict break-even, $E(x)=0$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Solve $x^6-12x^4+23x^2+36=0$. State all real and complex solutions.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Solve $x^3 - x^2 - 12x = 0$.	$x=0, x=4, x=-3$
2	Solve $x^3 + 6x^2 + 9x = 0$.	$x=0, x=-3$ (multiplicity 2)
3	Solve $x^4 - 36x^2 = 0$.	$x=0$ (multiplicity 2), $x=6, x=-6$
4	Solve $x^4 + 15x^2 - 16 = 0$.	$x=1, x=-1, x=4i, x=-4i$
5	Solve $x^3 - 8 = 0$.	$x=2, x=-1+i\sqrt{3}, x=-1-i\sqrt{3}$
6	Solve $x^4 - 10x^2 + 9 = 0$.	$x=1, x=-1, x=3, x=-3$

PART 2 • APPLY IT

- x=3 cm** Set $x(x+2)(x+4)=105$, then factor $x^3+6x^2+8x-105$ as $(x-3)(x^2+9x+35)$. Only $x=3$ is a positive real dimension.
- x=2 weeks and x=4 weeks** Factor the expression as $(x^2-6x+8)(x^2+9)=0$. The real roots are 2 and 4, while the other roots are $3i$ and $-3i$.

CHALLENGE

x=2, x=-2, x=3, x=-3, x=i, x=-i

Accept roots in any order and accept equivalent complex forms. For the context problems, accept only values that make sense in the stated situation.