LESSON MAKER PRO

Zero In

Use polynomial theorems to find remainders, factors, and zeros.

HIGH SCHOOL · MA.912.AR.6.3

MA.912.AR.6.3 "Explain and apply theorems for polynomials to solve mathematical and real-world problems."

NAME _____ DATE _____

FACTORS AND REMAINDERS

The Remainder Theorem says that the remainder when P(x) is divided by x - a is P(a). The Factor Theorem says x - a is a factor exactly when P(a) = 0. Use a zero to factor the polynomial, then solve the remaining factors.

WORKED EXAMPLE

Solve x^3 - 7x^2 + 14x - 8 = 0.

1. Test x = 1: P(1) = 0, so x - 1 is a factor.
2. Divide: x^3 - 7x^2 + 14x - 8 = (x - 1)(x^2 - 6x + 8).
3. Factor: x^2 - 6x + 8 = (x - 2)(x - 4).

Answer: x = 1, 2, or 4

PRACTICE 6 problems

1 Find the remainder when P(x) = x^3 + 5x^2 - 3x + 6 is divided by x + 3.

ANSWER _____

2 Is x - 5 a factor of P(x) = x^3 - 11x^2 + 3x + 135? Use the Factor Theorem.

ANSWER _____

3 Solve x^3 + 3x^2 - 18x = 0.

ANSWER _____

4 Solve x^3 - 9x = 0.

ANSWER _____

5 Find the quotient when x^3 - 3x^2 - 13x - 30 is divided by x - 6.

ANSWER _____

6 Find k if x + 5 is a factor of x^2 + kx - 55.

ANSWER _____

APPLY IT Show your work

1 An esports club models its profit from selling x dozens of event passes with P(x) = -5x^2 + 60x - 135, where P(x) is in dollars. At what values of x does the club break even? For what values of x is the profit positive?

WORK SPACE

ANSWER _____

2 A maker club builds a rectangular display case with dimensions x cm, x - 5 cm, and x - 6 cm. The case must have a volume of 330 cubic cm. Find x and the three dimensions.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Use the Rational Root Theorem to solve x^3 - 5x^2 - 36x + 180 = 0. Explain how you chose a value to test.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the remainder when P(x) = x^3 + 5x^2 - 3x + 6 is divided by x + 3.	**33**
2	Is x - 5 a factor of P(x) = x^3 - 11x^2 + 3x + 135? Use the Factor Theorem.	**Yes, because P(5) = 0.**
3	Solve x^3 + 3x^2 - 18x = 0.	**x = -6, 0, or 3**
4	Solve x^3 - 9x = 0.	**x = -3, 0, or 3**
5	Find the quotient when x^3 - 3x^2 - 13x - 30 is divided by x - 6.	**x^2 + 3x + 5**
6	Find k if x + 5 is a factor of x^2 + kx - 55.	**k = -6**

PART 2 · APPLY IT

1. **The club breaks even at x = 3 and x = 9. The profit is positive when 3 < x < 9.** Factor P(x) as -5(x - 3)(x - 9). The zeros give the break-even values, and the downward-opening parabola is positive between its zeros.
2. **x = 11 cm. The dimensions are 11 cm, 6 cm, and 5 cm.** Set x(x - 5)(x - 6) = 330 and rewrite as x^3 - 11x^2 + 30x - 330 = 0. Testing x = 11 gives a zero, and the remaining factor has no real zeros.

CHALLENGE

Possible whole-number rational zeros must be factors of 180. Testing x = 5 gives 0, so x - 5 is a factor. The quotient is x^2 - 36, so the solutions are x = -6, 5, and 6.

Accept equivalent factored forms and any correct use of synthetic division or polynomial division. For the word problems, require positive dimensions and interpret x as a positive quantity of dozens of passes.