

Polynomial Graph Detective

Sketch polynomial graphs and spot their key features.

HIGH SCHOOL • MA.912.AR.6.4

MA.912.AR.6.4 "Given a table, equation or written description of a polynomial function of degree 3 or higher, graph that function and determine its key features."

NAME _____

DATE _____

READ THE FACTORS

Zeros tell where a graph meets the x-axis. A zero with even multiplicity touches and turns, while a zero with odd multiplicity crosses. Use the degree and leading coefficient to find end behavior, and substitute $x = 0$ to find the y-intercept.

WORKED EXAMPLE

Sketch $f(x) = x^3 - 343$. Identify its intercepts and end behavior.

1. Set $f(x) = 0$: $x^3 = 343$, so $x = 7$.
2. Find the y-intercept: $f(0) = -343$.
3. The function is a cubic with a positive leading coefficient.
4. Plot $(7, 0)$ and $(0, -343)$, then draw a rising curve that crosses the x-axis.

Answer: x-intercept: $(7, 0)$; y-intercept: $(0, -343)$; left end down, right end up.

PRACTICE 5 problems

- 1** Sketch $y = (x - 1)(x + 2)(x - 4)$. List the x-intercepts, y-intercept, and end behavior.

ANSWER _____

- 2** Sketch $y = -(x + 3)^2(x - 1)$. List the x-intercepts, y-intercept, and end behavior.

ANSWER _____

- 3** Sketch $y = x^4 - 16$. List the intercepts and end behavior.

ANSWER _____

- 4** The simplest polynomial has degree 4, zeros $x = -4$ and $x = 3$, each with multiplicity 2, and a positive leading coefficient. Write and sketch the function. State its y-intercept and end behavior.

ANSWER _____

- 5** A cubic p has this table. Sketch it, find $p(x)$, and state its intercepts and end behavior. x : -3, -1, 0, 1, 3 $p(x)$: -24, 0, 0, 0, 24

ANSWER _____

APPLY IT Show your work

1

A student concert's net profit, in hundreds of dollars, is modeled by $P(x) = -(x - 1)(x - 4)(x + 2)$, where x is the dollar change from the planned ticket price. For $-3 \leq x \leq 5$, sketch the graph. At which price changes does the concert break even, and when does it make a profit?

WORK SPACE

ANSWER _____

2

A game designer models a character's height above a platform by $h(t) = (t - 1)^2(6 - t)$, where $0 \leq t \leq 6$. Sketch the graph on this interval. When is the character at the platform, and does the graph touch or cross at each time?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Without making a value table, sketch $y = (x + 2)^2(x - 1)^3$. Explain how multiplicity and the leading term determine the graph's behavior at each x -intercept and at both ends. Include the y -intercept.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Sketch $y = (x - 1)(x + 2)(x - 4)$. List the x-intercepts, y-intercept, and end behavior.	x-intercepts: $(-2, 0)$, $(1, 0)$, $(4, 0)$, all cross; y-intercept: $(0, 8)$; left end down, right end up.
2	Sketch $y = -(x + 3)^2(x - 1)$. List the x-intercepts, y-intercept, and end behavior.	x-intercepts: $(-3, 0)$, touches; $(1, 0)$, crosses; y-intercept: $(0, 9)$; left end up, right end down.
3	Sketch $y = x^4 - 16$. List the intercepts and end behavior.	x-intercepts: $(-2, 0)$ and $(2, 0)$, both cross; y-intercept: $(0, -16)$; both ends up.
4	The simplest polynomial has degree 4, zeros $x = -4$ and $x = 3$, each with multiplicity 2, and a positive leading coefficient. Write and sketch the function. State its y-intercept and end behavior.	$y = (x + 4)^2(x - 3)^2$; both x-intercepts touch; y-intercept: $(0, 144)$; both ends up.
5	A cubic p has this table. Sketch it, find $p(x)$, and state its intercepts and end behavior. x : $-3, -1, 0, 1, 3$ $p(x)$: $-24, 0, 0, 0, 24$	$p(x) = x^3 - x$; x-intercepts: $(-1, 0)$, $(0, 0)$, $(1, 0)$, all cross; y-intercept: $(0, 0)$; left end down, right end up.

PART 2 • APPLY IT

- Break-even price changes: $x = -2, 1$, and 4 . Profit occurs for $-3 \leq x < -2$ and $1 < x < 4$. The y-intercept is $(0, -8)$.** Set each factor equal to zero for break-even points. Use a test value in each interval to determine where $P(x)$ is positive.
- The character is at the platform at $t = 1$ and $t = 6$. The graph touches at $t = 1$ and crosses at $t = 6$ if the model is extended. The y-intercept is $(0, 6)$.** The factor $(t - 1)^2$ gives an even-multiplicity zero, so the graph touches. The factor $(6 - t)$ gives an odd-multiplicity zero, so the graph crosses.

CHALLENGE

x-intercepts: $(-2, 0)$, where the graph touches because the multiplicity is 2; and $(1, 0)$, where the graph crosses because the multiplicity is 3. **The y-intercept is $(0, -4)$. The degree is 5 with a positive leading coefficient, so the left end is down and the right end is up.**

Accept equivalent expanded or factored equations. Accept sketches that show the correct intercepts, touch or cross behavior, y-intercept, and end behavior.