

Polynomial Pathways

Use zeros, multiplicity, and end behavior to sketch graphs.

HIGH SCHOOL • MA.912.AR.6.5

MA.912.AR.6.5 "Sketch a rough graph of a polynomial function of degree 3 or higher using zeros, multiplicity and knowledge of end behavior."

NAME _____

DATE _____

BUILD THE SKETCH

A zero with odd multiplicity crosses the x-axis, while a zero with even multiplicity touches the x-axis and turns around. For end behavior, an even degree has matching ends and an odd degree has opposite ends. A positive leading coefficient rises on the right, and a negative leading coefficient falls on the right.

WORKED EXAMPLE

Sketch $f(x) = -(x-17)(x+19)^4$.

1. The zeros are $x=17$ and $x=-19$.
2. At $x=17$ the graph crosses. At $x=-19$ it touches and turns.
3. The degree is odd and the leading coefficient is negative, so the left end rises and the right end falls.

Answer: Touch at $x=-19$, cross at $x=17$, left end up, right end down.

PRACTICE 5 problems

- 1** Sketch $f(x) = (x+6)^2(x-8)$. Label each x-intercept and the end behavior.

ANSWER _____

- 2** Sketch $f(x) = -(x-7)^3(x+9)^2$. Label each x-intercept and the end behavior.

ANSWER _____

- 3** Sketch $f(x) = (x-10)(x+11)(x-12)^2$. Label each x-intercept and the end behavior.

ANSWER _____

- 4** Sketch $f(x) = -(x+15)^2(x-16)^2(x-20)$. Label each x-intercept and the end behavior.

ANSWER _____

- 5** Sketch $f(x) = (x-18)^5$. Label the x-intercept and the end behavior.

ANSWER _____

APPLY IT Show your work

- 1** A music app models an artist's buzz score by $B(t)=(t-21)^2(t-23)$, where t is days since a song release. Sketch the graph for $t \geq 0$. Mark the days when the score is zero, and describe the full polynomial's end behavior.

WORK SPACE

ANSWER _____

- 2** A gaming club models its sign-up trend by $C(t)=-(t-24)^3(t-26)^2$, where t is days after a tournament announcement. Sketch the graph for $t \geq 0$. Mark the zero days, and describe the full polynomial's end behavior.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Explain how to sketch $f(x)=-(x+28)^2(x-29)^3(x-30)$. State what happens at every zero, the end behavior, and whether the graph is above or below the x -axis between consecutive zeros.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Sketch $f(x)=(x+6)^2(x-8)$. Label each x-intercept and the end behavior.	Touch at $x=-6$, cross at $x=8$, left end down, right end up.
2	Sketch $f(x)=-(x-7)^3(x+9)^2$. Label each x-intercept and the end behavior.	Touch at $x=-9$, cross at $x=7$, left end up, right end down.
3	Sketch $f(x)=(x-10)(x+11)(x-12)^2$. Label each x-intercept and the end behavior.	Cross at $x=-11$ and $x=10$, touch at $x=12$, both ends up.
4	Sketch $f(x)=-(x+15)^2(x-16)^2(x-20)$. Label each x-intercept and the end behavior.	Touch at $x=-15$ and $x=16$, cross at $x=20$, left end up, right end down.
5	Sketch $f(x)=(x-18)^5$. Label the x-intercept and the end behavior.	Crosses and flattens at $x=18$, left end down, right end up.

PART 2 · APPLY IT

- For $t \geq 0$, the graph touches the axis at $t=21$ and crosses at $t=23$. The full graph has a left end down and a right end up.** The zero at 21 has even multiplicity, so it touches. The zero at 23 has odd multiplicity, and the positive cubic rises to the right.
- For $t \geq 0$, the graph crosses and flattens at $t=24$, then touches at $t=26$. The full graph has a left end up and a right end down.** The zero at 24 has odd multiplicity, so the graph crosses, and the zero at 26 has even multiplicity, so it touches. The negative odd-degree polynomial falls to the right.

CHALLENGE

The graph touches at $x=-28$, crosses and flattens at $x=29$, and crosses at $x=30$. Both ends are down. It is below the x-axis to the left of 29, including on both sides of -28, above the x-axis from 29 to 30, and below the x-axis to the right of 30.

Accept any rough sketch with the listed intercept behavior and end behavior. Small differences in curve shape are acceptable when crossings, touches, and relative above or below positions are correct.