

Rational Graph Routes

Use asymptotes, intercepts, and holes to sketch rational functions.

HIGH SCHOOL • MA.912.AR.8.2

MA.912.AR.8.2 "Given a table, equation or written description of a rational function, graph that function and determine its key features."

NAME _____

DATE _____

GRAPHING CLUES

Zeros of the denominator that do not cancel give vertical asymptotes. Compare the degrees for the horizontal asymptote, then find intercepts and any holes before sketching the branches.

WORKED EXAMPLE

Graph $f(x) = (x + 11)/(x - 13)$. State its key features.

1. Set $x - 13 = 0$, so the vertical asymptote is $x = 13$.
2. The degrees match, so the horizontal asymptote is $y = 1$.
3. Set $x + 11 = 0$ for the x-intercept, then find $f(0)$ for the y-intercept.

Answer: Vertical asymptote: $x = 13$.

Horizontal asymptote: $y = 1$. x-intercept: $(-11, 0)$. y-intercept: $(0, -11/13)$.

PRACTICE 5 problems

- 1** Graph $f(x) = (x - 2)/(x + 3)$. State the vertical asymptote, horizontal asymptote, and intercepts.

ANSWER _____

- 2** Graph $f(x) = 5/(x - 2)$. State the asymptotes and intercepts.

ANSWER _____

- 3** Graph $f(x) = (x^2 - 9)/(x - 3)$. State the hole, intercepts, and asymptote.

ANSWER _____

- 4** Graph $f(x) = (2x - 6)/(x + 5)$. State the vertical asymptote, horizontal asymptote, and intercepts.

ANSWER _____

- 5** Graph $f(x) = -2 - 6/(x - 6)$. State the vertical asymptote, horizontal asymptote, and intercepts.

ANSWER _____

APPLY IT Show your work

- 1** A gaming club pays a \$24 setup fee and \$6 each month for a server. Its average monthly cost after x months is $A(x) = (6x + 24)/x$. Graph A for meaningful values of x . State the relevant domain, asymptotes, and $A(6)$.

WORK SPACE

ANSWER _____

- 2** A video channel gets 60 launch-day views plus 12 views per hour. Its average views per hour after x hours is $V(x) = (12x + 60)/x$. Graph V for $x > 0$. State the asymptotes and find when the average is 24 views per hour.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Graph $f(x) = (x^2 - 4x - 12)/(x^2 - x - 6)$. State every key feature and explain why the graph has a hole instead of a vertical asymptote at $x = -2$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Graph $f(x) = (x - 2)/(x + 3)$. State the vertical asymptote, horizontal asymptote, and intercepts.	VA: $x = -3$. HA: $y = 1$. x-intercept: $(2, 0)$. y-intercept: $(0, -2/3)$.
2	Graph $f(x) = 5/(x - 2)$. State the asymptotes and intercepts.	VA: $x = 2$. HA: $y = 0$. No x-intercept. y-intercept: $(0, -5/2)$.
3	Graph $f(x) = (x^2 - 9)/(x - 3)$. State the hole, intercepts, and asymptote.	$f(x) = x + 3$, with x not equal to 3. Hole: $(3, 6)$. x-intercept: $(-3, 0)$. y-intercept: $(0, 3)$. Slant asymptote: $y = x + 3$.
4	Graph $f(x) = (2x - 6)/(x + 5)$. State the vertical asymptote, horizontal asymptote, and intercepts.	VA: $x = -5$. HA: $y = 2$. x-intercept: $(3, 0)$. y-intercept: $(0, -6/5)$.
5	Graph $f(x) = -2 - 6/(x - 6)$. State the vertical asymptote, horizontal asymptote, and intercepts.	VA: $x = 6$. HA: $y = -2$. x-intercept: $(3, 0)$. y-intercept: $(0, -1)$.

PART 2 • APPLY IT

- Relevant domain: $x > 0$. VA: $x = 0$. HA: $y = 6$. $A(6) = 10$, so the average cost is \$10 per month.** Rewrite $A(x)$ as $6 + 24/x$. The setup fee is spread across more months, so the average approaches \$6.
- VA: $x = 0$. HA: $y = 12$. The average is 24 views per hour when $x = 5$.** Rewrite $V(x)$ as $12 + 60/x$. Solve $12 + 60/x = 24$ to get $x = 5$.

CHALLENGE

Factor to get $f(x) = (x - 6)(x + 2)/[(x - 3)(x + 2)] = (x - 6)/(x - 3)$, with x not equal to -2 or 3. Hole: $(-2, 8/5)$. VA: $x = 3$. HA: $y = 1$. x-intercept: $(6, 0)$. y-intercept: $(0, 2)$. The factor $x + 2$ cancels, so $x = -2$ is excluded but does not create a vertical asymptote.

Accept equivalent simplified forms and correctly labeled sketches with the listed features. For contextual graphs, accept only the branch with $x > 0$ as the relevant domain.