

Piecewise Pathways

Choose the rule, graph the pieces, and use context constraints.

HIGH SCHOOL • MA.912.AR.9.10

MA.912.AR.9.10 "Solve and graph mathematical and real-world problems that are modeled with piecewise functions. Interpret key features and determine constraints in terms of the context."

NAME _____

DATE _____

ONE INPUT, ONE RULE

For each x-value, use only the rule whose condition includes that value. To graph a piecewise function, draw each rule only on its stated domain, using an open circle for an excluded endpoint and a closed circle for an included endpoint.

WORKED EXAMPLE

Given $f(x) = \{2x + 1 \text{ if } x < 4; 9 \text{ if } x \geq 4\}$, find $f(4)$.

1. Since $4 \geq 4$, use $f(x) = 9$.

2. Substitute: $f(4) = 9$.

Answer: $f(4) = 9$

PRACTICE 6 problems

1 Given $f(x) = \{3x - 5 \text{ if } x < 6; x + 7 \text{ if } x \geq 6\}$, find $f(6)$.

ANSWER _____

2 Solve $g(x) = 17$ for $g(x) = \{3x + 8 \text{ if } x \leq 3; 17 \text{ if } x > 3\}$.

ANSWER _____

3 Graph $h(x) = \{-x + 11 \text{ if } x < 5; 6 \text{ if } x \geq 5\}$. State the open circle and closed circle.

ANSWER _____

4 Find the domain and range of $q(x) = \{7x \text{ if } x < 0; 12 \text{ if } x \geq 0\}$.

ANSWER _____

5 Solve $r(x) = 0$ for $r(x) = \{15 - 3x \text{ if } x < 7; x - 6 \text{ if } x \geq 7\}$.

ANSWER _____

6 Is $p(x) = \{x + 10 \text{ if } x < 8; 3x - 6 \text{ if } x \geq 8\}$ continuous at $x = 8$? Explain with the values from both rules.

ANSWER _____

APPLY IT Show your work

- 1** A concert venue charges \$7 to park for up to 3 hours. After 3 hours, it charges \$5 per additional hour, with partial hours charged proportionally. Parking time x must satisfy $0 < x \leq 8$. Write a piecewise function $C(x)$ for the cost, find $C(6)$, and graph the function.

WORK SPACE

ANSWER _____

- 2** A school club sells custom stickers. Each of the first 5 stickers costs \$6. Each sticker after 5 costs \$3, and the club can order no more than 11 stickers. Let x be the whole-number number of stickers ordered. Write a piecewise function $S(x)$, state the constraints, and find $S(7)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A gaming lounge charges a \$5 entry fee that covers a session up to 3 hours. From more than 3 hours through 6 hours, it adds \$7 per hour beyond 3. After 6 hours, it adds \$6 per hour beyond 6. A session time x must satisfy $0 < x \leq 8$. Write a piecewise function $L(x)$, graph it, find $L(7)$, and explain why the domain conditions do not overlap.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given $f(x) = \{3x - 5 \text{ if } x < 6; x + 7 \text{ if } x \geq 6\}$, find $f(6)$.	$f(6) = 13$
2	Solve $g(x) = 17$ for $g(x) = \{3x + 8 \text{ if } x \leq 3; 17 \text{ if } x > 3\}$.	$x \geq 3$
3	Graph $h(x) = \{-x + 11 \text{ if } x < 5; 6 \text{ if } x \geq 5\}$. State the open circle and closed circle.	Graph $y = -x + 11$ for $x < 5$ with an open circle at $(5, 6)$. Graph $y = 6$ for $x \geq 5$ with a closed circle at $(5, 6)$.
4	Find the domain and range of $q(x) = \{7x \text{ if } x < 0; 12 \text{ if } x \geq 0\}$.	Domain: all real numbers. Range: $y < 0$ or $y = 12$.
5	Solve $r(x) = 0$ for $r(x) = \{15 - 3x \text{ if } x < 7; x - 6 \text{ if } x \geq 7\}$.	$x = 5$
6	Is $p(x) = \{x + 10 \text{ if } x < 8; 3x - 6 \text{ if } x \geq 8\}$ continuous at $x = 8$? Explain with the values from both rules.	Yes. The first rule gives $8 + 10 = 18$, and the second rule gives $3(8) - 6 = 18$, so the pieces meet at $(8, 18)$.

PART 2 · APPLY IT

- $C(x) = \{7 \text{ if } 0 < x \leq 3; 7 + 5(x - 3) \text{ if } 3 < x \leq 8\}$. $C(6) = 22$. Graph $y = 7$ for $0 < x \leq 3$, then graph $y = 7 + 5(x - 3)$ for $3 < x \leq 8$. Use a closed point at $(3, 7)$ and an endpoint at $(8, 32)$.** Use the flat fee through 3 hours. For 6 hours, add 5 dollars for each of the 3 hours beyond 3.
- $S(x) = \{6x \text{ if } 0 < x \leq 5; 30 + 3(x - 5) \text{ if } 5 < x \leq 11\}$. Constraints: x is a whole number and $0 < x \leq 11$. $S(7) = 36$.** The first 5 stickers cost 30 dollars. For 7 stickers, add the cost of the 2 stickers beyond 5.

CHALLENGE

$L(x) = \{5 \text{ if } 0 < x \leq 3; 5 + 7(x - 3) \text{ if } 3 < x \leq 6; 26 + 6(x - 6) \text{ if } 6 < x \leq 8\}$. $L(7) = 32$. Graph $y = 5$ through $(3, 5)$, then graph the line from $(3, 5)$ to $(6, 26)$, then graph the line from $(6, 26)$ to $(8, 38)$. The conditions do not overlap because $x = 3$ belongs only to the first rule and $x = 6$ belongs only to the second rule.

Accept equivalent simplified expressions, such as $5x - 8$ for the second parking rule. Accept correctly labeled graphs with equivalent endpoint notation, provided open and closed circles match the stated inequalities.