

Shade the Overlap

Graph systems of inequalities and find the shared region.

HIGH SCHOOL • MA.912.AR.9.5

MA.912.AR.9.5 "Graph the solution set of a system of two-variable inequalities."

NAME _____

DATE _____

FIND THE SHARED REGION

Graph each inequality boundary first. Use a solid line for $\leq$ or $\geq$, and use a dashed line for $<$ or $>$. Shade the side that makes each inequality true, then keep only the region where all shadings overlap.

WORKED EXAMPLE

Graph the system: $y \geq 3x - 5$ and $y < -2x + 8$.

1. Graph $y = 3x - 5$ as a solid line.
2. Graph $y = -2x + 8$ as a dashed line.
3. Set $3x - 5 = -2x + 8$. Then $5x = 13$, so the lines meet at $(13/5, 14/5)$.
4. Shade above or on the solid line and below the dashed line.

Answer: The solution is the region above or on $y = 3x - 5$ and below $y = -2x + 8$. The lines meet at $(13/5, 14/5)$, but that point is not included because $y < -2x + 8$ is strict.

PRACTICE 6 problems

1 Graph the system: $y \leq x + 4$ and $y > -x + 6$.

ANSWER _____

2 Graph the system: $x + y \geq 7$ and $x - y < 1$.

ANSWER _____

3 Graph the system: $x \geq 4$ and $y < 6$.

ANSWER _____

4 Graph the system: $y \geq 4x + 7$ and $y < 4x + 9$.

ANSWER _____

5 Graph the system: $y \leq 0$ and $x + y > 9$.

ANSWER _____

6 Graph the system: $x \geq 0$, $y \geq 0$, and $x + y \leq 7$.

ANSWER _____

APPLY IT Show your work

- 1** For a club sale, you make x custom stickers and y pin buttons. A sticker takes 4 minutes to make, a pin takes 1 minute, and you have at most 36 minutes. You need at least 9 items. Write and graph a system, including nonnegative amounts.

WORK SPACE

ANSWER _____

- 2** A school concert sells x standard tickets for \$6 each and y VIP tickets for \$9 each. The club can sell at most 10 tickets and needs at least \$54 in ticket revenue. Write and graph a system, including nonnegative amounts.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Graph the system: $y < x + 4$, $y < -x + 6$, and $y \geq 0$. Identify each boundary type and explain why none of the three corner points is included.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Graph the system: $y \leq x + 4$ and $y > -x + 6$.	Draw $y = x + 4$ solid and shade below it. Draw $y = -x + 6$ dashed and shade above it. The overlap is between the lines to the right of their intersection, $(1, 5)$, and $(1, 5)$ is not included.
2	Graph the system: $x + y \geq 7$ and $x - y < 1$.	Rewrite as $y \geq -x + 7$ and $y > x - 1$. Draw $y = -x + 7$ solid and $y = x - 1$ dashed, then shade above both lines. They meet at $(4, 3)$, which is not included.
3	Graph the system: $x \geq 4$ and $y < 6$.	Draw $x = 4$ as a solid vertical line and shade right. Draw $y = 6$ as a dashed horizontal line and shade below. The solution is the overlap.
4	Graph the system: $y \geq 4x + 7$ and $y < 4x + 9$.	Draw $y = 4x + 7$ solid and $y = 4x + 9$ dashed. Shade above or on the lower line and below the upper line. The solution is the strip between the parallel lines.
5	Graph the system: $y \leq 0$ and $x + y > 9$.	Rewrite $x + y > 9$ as $y > -x + 9$. Draw $y = 0$ solid and $y = -x + 9$ dashed. Shade below or on $y = 0$ and above $y = -x + 9$. The lines meet at $(9, 0)$, which is not included.
6	Graph the system: $x \geq 0$, $y \geq 0$, and $x + y \leq 7$.	Draw all three boundaries as solid lines. Shade the first-quadrant triangle below $x + y = 7$, including its edges. Its vertices are $(0, 0)$, $(7, 0)$, and $(0, 7)$.

PART 2 · APPLY IT

- $4x + y \leq 36$, $x + y \geq 9$, $x \geq 0$, $y \geq 0$. Graph the first-quadrant region below or on $4x + y = 36$ and above or on $x + y = 9$. The boundary vertices are $(0, 9)$, $(0, 36)$, and $(9, 0)$. Use total making time for $4x + y \leq 36$ and total items for $x + y \geq 9$. The feasible graph is the overlap in the first quadrant, with all boundaries solid.
- $x + y \leq 10$, $6x + 9y \geq 54$, $x \geq 0$, $y \geq 0$. Graph the first-quadrant region below or on $x + y = 10$ and above or on $6x + 9y = 54$. The boundary vertices are $(9, 0)$, $(10, 0)$, $(0, 10)$, and $(0, 6)$. Use the ticket limit for $x + y \leq 10$ and the revenue requirement for $6x + 9y \geq 54$. Both inequalities include equality, so all boundary lines are solid.

CHALLENGE

Draw $y = x + 4$ and $y = -x + 6$ as dashed lines. Draw $y = 0$ as a solid line. Shade the triangular region above or on $y = 0$ and below both dashed lines. The corner points are $(-4, 0)$, $(6, 0)$, and $(1, 5)$, and none is included because each lies on at least one dashed boundary.

Accept equivalent inequality forms and correct graphs with the same boundary types and overlap region. In the two contexts, x and y represent item or ticket counts, so actual choices are nonnegative whole-number points in the graphed region.