Corner Point Decisions

Use constraints and corner points to make the best choice.

HIGH SCHOOL · MA.912.AR.9.8

MA.912.AR.9.8 "Solve real-world problems involving linear programming in two variables."

NAME ____________________ DATE ____________

FIND THE BEST CORNER

For a linear programming problem, use the inequalities to describe the feasible region, including x >= 0 and y >= 0 when the quantities cannot be negative. A maximum or minimum value occurs at a corner point, so evaluate the objective expression at every feasible corner point.

WORKED EXAMPLE

Maximize P = 41x + 29y subject to x + y <= 31, 23x + y <= 317, x >= 0, and y >= 0.

1. Find the boundary intersection: x + y = 31 and 23x + y = 317, so x = 13 and y = 18.
2. Include the axis corners: (0,0), (317/23,0), and (0,31).
3. Evaluate P at the corners: 0, 12997/23, 1055, and 899.
4. The largest value is 1055 at (13,18).

Answer: Maximum P = 1055 when x = 13 and y = 18.

PRACTICE

6 problems

1 Maximize P = 6x + 5y subject to x + y <= 9, 5x + y <= 33, x >= 0, and y >= 0.

ANSWER ____________

2 Maximize C = 8x + 9y subject to x + y <= 6, x + 5y <= 26, x >= 0, and y >= 0.

ANSWER ____________

3 A feasible region has corner points (0,0), (9,0), (5,6), and (0,8). Maximize P = 5x + 6y.

ANSWER ____________

4 Minimize C = 8x + 5y subject to x + y >= 9, 5x + y >= 20, x >= 0, and y >= 0.

ANSWER ____________

5 Maximize R = 9x + 8y subject to 6x + y <= 36, x + 6y <= 36, x >= 0, and y >= 0.

ANSWER ____________

6 A feasible region has corner points (0,0), (0,8), (6,8), (9,5), and (9,0). Maximize D = 6x + 9y.

ANSWER ____________

APPLY IT Show your work

1 At a campus craft sale, a student can make tote bags, x, and patches, y. The student can make no more than 16 total items, and machine capacity is modeled by $5x + y <= 48$. Each tote bag earns \$12 profit, and each patch earns \$8 profit. How many of each item should the student make to maximize profit?

WORK SPACE

ANSWER ______________________

2 A student council is buying snack boxes for an event. A granola box, x, provides 5 protein servings and 1 fruit serving and costs \$9. A fruit box, y, provides 1 protein serving and 1 fruit serving and costs \$5. The council needs at least 21 protein servings and at least 9 fruit servings. Write a system of inequalities and find the least possible cost.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

A student is choosing nonnegative whole-number quantities x and y for two types of fundraiser items. Maximize $R = 9x + 12y$ subject to $x + y <= 14$, $5x + 3y <= 54$, and $3x + 5y <= 54$. Find the maximum revenue and explain how you know your whole-number solution is best.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Maximize P = 6x + 5y subject to x + y <= 9, 5x + y <= 33, x >= 0, and y >= 0.	**Maximum P = 51 when x = 6 and y = 3.**
2	Maximize C = 8x + 9y subject to x + y <= 6, x + 5y <= 26, x >= 0, and y >= 0.	**Maximum C = 53 when x = 1 and y = 5.**
3	A feasible region has corner points (0,0), (9,0), (5,6), and (0,8). Maximize P = 5x + 6y.	**Maximum P = 61 at (5,6).**
4	Minimize C = 8x + 5y subject to x + y >= 9, 5x + y >= 20, x >= 0, and y >= 0.	**Minimum C = 213/4 when x = 11/4 and y = 25/4.**
5	Maximize R = 9x + 8y subject to 6x + y <= 36, x + 6y <= 36, x >= 0, and y >= 0.	**Maximum R = 612/7 when x = 36/7 and y = 36/7.**
6	A feasible region has corner points (0,0), (0,8), (6,8), (9,5), and (9,0). Maximize D = 6x + 9y.	**Maximum D = 108 at (6,8).**

PART 2 · APPLY IT

1. **Make 8 tote bags and 8 patches. The maximum profit is $160.** Use x + y <= 16 and 5x + y <= 48, then maximize P = 12x + 8y. The corner point (8,8) gives P = 160, which is greater than the values at the other corners.
2. **System: 5x + y >= 21, x + y >= 9, x >= 0, y >= 0. Buy 3 granola boxes and 6 fruit boxes for a minimum cost of $57.** Minimize C = 9x + 5y. The boundary lines intersect at (3,6), and C = 9(3) + 5(6) = 57, less than the cost at the other feasible corners.

CHALLENGE

The maximum revenue is R = 138 at x = 6 and y = 7. For each whole-number value of y from 0 through 10, use the constraints to find the greatest possible whole-number x, since revenue increases when x increases. The largest resulting value is 9(6) + 12(7) = 138.

Accept equivalent objective-value forms, including 53.25 for 213/4. Students may use a graph, a table of corner points, or algebraic boundary intersections, but their chosen point must satisfy every constraint.