

Three Unknowns, Solved

Use elimination to solve systems with three variables.

HIGH SCHOOL • MA.912.AR.9.9

MA.912.AR.9.9 "Given a mathematical or real-world context, solve a system of three-variable linear equations algebraically."

NAME _____

DATE _____

ELIMINATE, THEN SOLVE

Use elimination to remove the same variable from two pairs of equations. Solve the resulting two-variable system, then substitute back to find the third variable and check all three equations.

WORKED EXAMPLE

Solve the system: $x + y + z = 41$
 $x - y + z = 19$
 $x + y - z = 13$

1. Subtract the second equation from the first: $2y = 22$.
2. Divide by 2: $y = 11$.
3. Subtract the third equation from the first: $2z = 28$, so $z = 14$.
4. Substitute into the first equation: $x = 16$.

Answer: $x = 16, y = 11, z = 14$

PRACTICE 6 problems

- 1** Solve the system algebraically: $x + y + z = 46$
 $x - y + z = 20$
 $x + y - z = 16$

ANSWER _____

- 2** Solve the system algebraically: $x + y + z = 58$
 $x - y + z = 34$
 $x + y - z = 24$

ANSWER _____

- 3** Solve the system algebraically: $2x + y + z = 52$
 $x + 2y + z = 59$
 $x + y + 2z = 65$

ANSWER _____

- 4** Solve the system algebraically: $3x + y - z = 15$
 $x + 3y + z = 25$
 $x - y + 3z = 23$

ANSWER _____

- 5** Solve the system algebraically: $2x - y + z = 20$
 $x + 2y - z = 4$
 $x + y + 2z = 28$

ANSWER _____

- 6** Solve the system algebraically: $x + y + z = 18$
 $2x - y + z = 17$
 $x + 2y - z = 8$

ANSWER _____

APPLY IT Show your work

1

At a school esports fundraiser, a game pass costs p dollars, a snack voucher costs s dollars, and a raffle ticket costs r dollars. Three game passes, two snack vouchers, and one raffle ticket cost \$25. Two game passes, one snack voucher, and two raffle tickets cost \$25. One game pass, three snack vouchers, and two raffle tickets cost \$27. Write and solve a system to find each price.

WORK SPACE

ANSWER _____

2

At a pop-up school store, a cap costs c dollars, a shirt costs h dollars, and a sticker pack costs s dollars. Two caps, one shirt, and three sticker packs cost \$35. One cap, three shirts, and two sticker packs cost \$40. Three caps, two shirts, and one sticker pack cost \$39. Write and solve a system to find each price.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Without graphing, solve the system. Then explain why your elimination work shows that the system has only one solution. $4x - 2y + z = 11$ $3x + y - 2z = 5$ $2x + 3y + 4z = 24$

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve the system algebraically: $x + y + z = 46$ $x - y + z = 20$ $x + y - z = 16$	$x = 18, y = 13, z = 15$
2	Solve the system algebraically: $x + y + z = 58$ $x - y + z = 34$ $x + y - z = 24$	$x = 29, y = 12, z = 17$
3	Solve the system algebraically: $2x + y + z = 52$ $x + 2y + z = 59$ $x + y + 2z = 65$	$x = 8, y = 15, z = 21$
4	Solve the system algebraically: $3x + y - z = 15$ $x + 3y + z = 25$ $x - y + 3z = 23$	$x = 6, y = 4, z = 7$
5	Solve the system algebraically: $2x - y + z = 20$ $x + 2y - z = 4$ $x + y + 2z = 28$	$x = 7, y = 3, z = 9$
6	Solve the system algebraically: $x + y + z = 18$ $2x - y + z = 17$ $x + 2y - z = 8$	$x = 7, y = 4, z = 7$

PART 2 · APPLY IT

- $p = \$4, s = \$3, r = \$7$** Use the three purchase totals to write $3p + 2s + r = 25$, $2p + s + 2r = 25$, and $p + 3s + 2r = 27$. Eliminate variables until one price is found, then substitute back.
- $c = \$6, h = \$8, s = \$5$** Write $2c + h + 3s = 35$, $c + 3h + 2s = 40$, and $3c + 2h + s = 39$. Eliminate one variable from two pairs of equations, solve the resulting two-variable system, and substitute back.

CHALLENGE

$x = 3, y = 2, z = 3$. Eliminating x gives $-10y + 11z = 13$ and $8y + 7z = 37$. Eliminating y gives $79z = 237$, so z has one value, then y and x each have one value.

Accept any algebraically equivalent elimination or substitution method. For word problems, accept prices written with or without dollar signs when the variable values are correct.