LESSON MAKER PRO

Between and Beyond

Use continuity to prove values exist and extrema occur.

HIGH SCHOOL · MA.912.C.1.11

MA.912.C.1.11 "Apply the Intermediate Value Theorem and the Extreme Value Theorem."

NAME _____ DATE _____

CONTINUITY GUARANTEES

If a function is continuous on a closed interval, the Intermediate Value Theorem guarantees every y-value between two function values occurs at least once. The Extreme Value Theorem guarantees an absolute maximum and an absolute minimum on that interval. Check continuity, then compare endpoint values and any critical-point values when finding extrema.

WORKED EXAMPLE

Let $g(x)=x^2+7$ on $[2, 6]$. Use the Intermediate Value Theorem to show that $g(c)=20$ for some c in $(2, 6)$.

1. g is a polynomial, so it is continuous on [2, 6].
2. g(2)=11 and g(6)=43.
3. The value 20 lies between 11 and 43.
4. By the Intermediate Value Theorem, g(c)=20 for some c in (2, 6).

Answer: A value c in (2, 6) exists such that g(c)=20.

PRACTICE 6 problems

1 $f(x)=x^3-4x+1$ is continuous on $[-3, 5]$. Use the Intermediate Value Theorem to decide whether $f(c)=8$ for some c in $(-3, 5)$.

ANSWER _____

2 A function h is continuous on [-4, 1], with h(-4)=9 and h(1)=-8. What does the Intermediate Value Theorem guarantee about h(c)=3?

ANSWER _____

3 Find the absolute maximum and absolute minimum of $p(x)=x^2-10x+12$ on $[1, 9]$.

ANSWER _____

4 Does the Extreme Value Theorem guarantee an absolute maximum and minimum for $q(x)=1/(x-4)$ on $[3, 5]$? Explain.

ANSWER _____

5 $r(x)=|x+3|$ is continuous on $[-5, 1]$. Use the Intermediate Value Theorem, then solve, to find c such that $r(c)=3$.

ANSWER _____

6 Find the absolute maximum and absolute minimum of $s(x)=x^3-48x$ on $[-5, 5]$.

ANSWER _____

APPLY IT Show your work

1 A gaming laptop's battery charge B(t), measured as a percent, changes continuously from B(1)=92 to B(5)=36. Must there be a time when the charge is 60 percent? State the interval where that time occurs.

WORK SPACE

ANSWER _____

2 For a robotics club launch, a model rocket's height is $h(t)=-t^2+10t+3$, where t is in seconds and $1<=t<=9$. Find the rocket's greatest height during this time interval.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A function f is continuous on [-8, 8]. You know that f(-8)=15, f(-3)=-9, f(1)=18, and f(8)=4. Explain what the Intermediate Value Theorem guarantees about solutions to f(x)=0. Then state what the Extreme Value Theorem guarantees on [-8, 8].

WORK SPACE AND ANSWER

ANSWER KEY # Between and Beyond

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	$f(x)=x^3-4x+1$ is continuous on [-3, 5]. Use the Intermediate Value Theorem to decide whether f(c)=8 for some c in (-3, 5).	**Yes. f(-3)=-14 and f(5)=106, and 8 is between these values. Therefore, some c in (-3, 5) has f(c)=8.**
2	A function h is continuous on [-4, 1], with h(-4)=9 and h(1)=-8. What does the Intermediate Value Theorem guarantee about h(c)=3?	**There is at least one c in (-4, 1) such that h(c)=3.**
3	Find the absolute maximum and absolute minimum of $p(x)=x^2-10x+12$ on [1, 9].	**Absolute maximum: 3 at x=1 and x=9. Absolute minimum: -13 at x=5.**
4	Does the Extreme Value Theorem guarantee an absolute maximum and minimum for q(x)=1/(x-4) on [3, 5]? Explain.	**No. q is not continuous on [3, 5] because it is undefined at x=4.**
5	r(x)=|x+3| is continuous on [-5, 1]. Use the Intermediate Value Theorem, then solve, to find c such that r(c)=3.	**c=0.**
6	Find the absolute maximum and absolute minimum of $s(x)=x^3-48x$ on [-5, 5].	**Absolute maximum: 128 at x=-4. Absolute minimum: -128 at x=4.**

PART 2 · APPLY IT

1. **Yes. There is a time c in (1, 5) when B(c)=60.** Because B is continuous and 60 is between 92 and 36, the Intermediate Value Theorem guarantees a time c in (1, 5).
2. **The greatest height is 28 meters at t=5 seconds.** The height function is continuous, so the Extreme Value Theorem applies. The vertex occurs at t=5, where h(5)=28, and both endpoint heights are 12.

CHALLENGE

There are at least two distinct solutions to f(x)=0: one in (-8, -3) and one in (-3, 1). The Extreme Value Theorem guarantees that f has an absolute maximum and an absolute minimum somewhere on [-8, 8].

For Intermediate Value Theorem items, accept any correct statement that identifies existence in the required open interval. For extrema items, require correct values and all x-values where each absolute extremum occurs.