

Holes and Horizons

Factor through a missing value.

HIGH SCHOOL • MA.912.C.1.3

MA.912.C.1.3 "Find limits of rational functions that are undefined at a point."

NAME _____

DATE _____

FIND THE MISSING LIMIT

If substituting the target x-value gives $0/0$, factor the numerator and denominator. Cancel only factors, then substitute into the simplified expression to find the limit. The original function can be undefined at the target even when the limit exists.

WORKED EXAMPLE

Find $\lim$ as x approaches 7 of $(x^2 - 49)/(x - 7)$.

1. Substitution gives $(49 - 49)/(7 - 7) = 0/0$.
2. Factor the numerator: $x^2 - 49 = (x - 7)(x + 7)$.
3. Cancel $(x - 7)$, so the expression is $x + 7$ for x not equal to 7.
4. Substitute 7: $7 + 7 = 14$.

Answer: 14

PRACTICE 6 problems

- 1** Find $\lim$ as x approaches 2 of $(x^2 - 4)/(x - 2)$.

ANSWER _____

- 2** Find $\lim$ as x approaches -3 of $(x^2 + 5x + 6)/(x + 3)$.

ANSWER _____

- 3** Find $\lim$ as x approaches 4 of $(x^2 - 16)/(x - 4)$.

ANSWER _____

- 4** Find $\lim$ as x approaches -1 of $(x^2 - 1)/(x + 1)$.

ANSWER _____

- 5** Find $\lim$ as x approaches 3 of $(x^2 - 7x + 12)/(x - 3)$.

ANSWER _____

- 6** Find $\lim$ as x approaches -2 of $(x^3 + 8)/(x + 2)$.

ANSWER _____

APPLY IT Show your work

- 1** A student models the storage used by short videos with $S(x) = x^2$ gigabytes after x weeks. The average change in storage from week 4 to week x is $(x^2 - 16)/(x - 4)$. Find the limit as x approaches 4. What does the limit mean in this model?

WORK SPACE

ANSWER _____

- 2** An esports team models its total points after x matches with $P(x) = x^2 + 4x$. Its average points gained per match from match 2 to match x is $(x^2 + 4x - 12)/(x - 2)$. Find the limit as x approaches 2. What does the limit estimate?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find $\lim$ as x approaches 2 of $(x^2 - 5x + 6)/(x^2 - 4)$. Show the factors you cancel.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\lim$ as x approaches 2 of $(x^2 - 4)/(x - 2)$.	4
2	Find $\lim$ as x approaches -3 of $(x^2 + 5x + 6)/(x + 3)$.	-1
3	Find $\lim$ as x approaches 4 of $(x^2 - 16)/(x - 4)$.	8
4	Find $\lim$ as x approaches -1 of $(x^2 - 1)/(x + 1)$.	-2
5	Find $\lim$ as x approaches 3 of $(x^2 - 7x + 12)/(x - 3)$.	-1
6	Find $\lim$ as x approaches -2 of $(x^3 + 8)/(x + 2)$.	12

PART 2 · APPLY IT

- 8 GB per week. The model predicts that storage use is increasing at 8 GB per week at week 4.** Factor $x^2 - 16$ as $(x - 4)(x + 4)$, then cancel $x - 4$. Evaluate $x + 4$ at $x = 4$.
- 8 points per match. The limit estimates the team's points-gaining rate at match 2 in the model.** Factor the numerator as $(x - 2)(x + 6)$, then cancel $x - 2$. Evaluate $x + 6$ at $x = 2$.

CHALLENGE

-1/4

Accept equivalent simplified expressions and correct factorization work. For the word problems, accept equivalent statements of the correct rate with units; for the challenge, accept -1/4 with valid cancellation of $x - 2$.