

Approach From One Side

Read the behavior from the left or right.

HIGH SCHOOL • MA.912.C.1.4

MA.912.C.1.4 "Find one-sided limits."

NAME _____

DATE _____

ONE SIDE AT A TIME

A left-hand limit looks at x -values less than the target value. A right-hand limit looks at x -values greater than the target value. Use only the rule, pattern, or graph behavior from the requested side.

WORKED EXAMPLE

Find $\lim_{x \rightarrow 7^-} (x^2 - 49)/(x - 7)$.

1. Factor the numerator: $x^2 - 49 = (x - 7)(x + 7)$.
2. For x not equal to 7, the expression simplifies to $x + 7$.
3. As x approaches 7 from the left, $x + 7$ approaches 14.

Answer: 14

PRACTICE 6 problems

- 1** For $f(x) = x + 5$ if $x < 3$, and $f(x) = 4x - 1$ if $x \geq 3$, find $\lim_{x \rightarrow 3^-} f(x)$.

ANSWER _____

- 2** For $g(x) = 6 - x$ if $x \leq -4$, and $g(x) = x + 9$ if $x > -4$, find $\lim_{x \rightarrow -4^+} g(x)$.

ANSWER _____

- 3** Find $\lim_{x \rightarrow 0^-} |x|/x$.

ANSWER _____

- 4** Find $\lim_{x \rightarrow -3^+} ((x + 3)(x - 5))/(x + 3)$.

ANSWER _____

- 5** Let $p(x)$ be the greatest integer less than or equal to x . Find $\lim_{x \rightarrow 5^-} p(x)$.

ANSWER _____

- 6** Find $\lim_{x \rightarrow 6^-} 1/(x - 6)$.

ANSWER _____

APPLY IT Show your work

- 1** A game gives bonus points $B(x)$, where x is a player's score level. $B(x) = x + 9$ for $x < 10$, and $B(x) = 3x - 11$ for $x \geq 10$. Find $\lim_{x \rightarrow 10^-} B(x)$. What does this mean in context?

WORK SPACE

ANSWER _____

- 2** An online art contest displays a featured score $R(t)$, where t is the number of hours after entries open. $R(t) = 5t - 8$ for $t < 4$, and $R(t) = 3t + 3$ for $t \geq 4$. Find $\lim_{t \rightarrow 4^+} R(t)$. What does this mean in context?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $h(x) = ((x - 5)(x + 4))/(x - 5)$ if $x < 5$, and $h(x) = 3x - 6$ if $x \geq 5$, find $\lim_{x \rightarrow 5^-} h(x)$, $\lim_{x \rightarrow 5^+} h(x)$, and $\lim_{x \rightarrow 5} h(x)$. Explain how you know the two-sided limit exists.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x + 5$ if $x < 3$, and $f(x) = 4x - 1$ if $x \geq 3$, find $\lim_{x \rightarrow 3^-} f(x)$.	8
2	For $g(x) = 6 - x$ if $x \leq -4$, and $g(x) = x + 9$ if $x > -4$, find $\lim_{x \rightarrow -4^+} g(x)$.	5
3	Find $\lim_{x \rightarrow 0^-} x /x$.	-1
4	Find $\lim_{x \rightarrow -3^+} ((x + 3)(x - 5))/(x + 3)$.	-8
5	Let $p(x)$ be the greatest integer less than or equal to x . Find $\lim_{x \rightarrow 5^-} p(x)$.	4
6	Find $\lim_{x \rightarrow 6^-} 1/(x - 6)$.	-infinity

PART 2 · APPLY IT

- 19; as the score level gets closer to 10 from below, the bonus points get closer to 19.** Use the $x < 10$ rule because the score approaches 10 from the left. Substitute 10 into $x + 9$.
- 15; just after 4 hours, the featured score gets closer to 15.** Use the $t \geq 4$ rule because time approaches 4 from the right. Substitute 4 into $3t + 3$.

CHALLENGE

$\lim_{x \rightarrow 5^-} h(x) = 9$, $\lim_{x \rightarrow 5^+} h(x) = 9$, and $\lim_{x \rightarrow 5} h(x) = 9$ because the one-sided limits are equal.

Accept equivalent limit notation, including $-\infty$ for -infinity. In the challenge, require both one-sided limits and the conclusion that the two-sided limit exists because they match.