

Infinity End Behavior

Find what rational expressions approach as x grows without bound.

HIGH SCHOOL • MA.912.C.1.5

MA.912.C.1.5 "Find limits at infinity."

NAME _____

DATE _____

FOCUS ON LEADING TERMS

To find a limit at infinity for a rational expression, compare the terms with the highest powers of x . If the degrees match, use the ratio of the leading coefficients. If the numerator has a smaller degree, the limit is 0, and if it has a larger degree, use the leading terms to determine whether the expression grows positively or negatively.

WORKED EXAMPLE

Find $\lim$ as x approaches infinity of $(11x^4 - 3x + 8)/(9x^4 + 5)$.

1. Divide the numerator and denominator by x^4 .
2. $(11 - 3/x^3 + 8/x^4)/(9 + 5/x^4)$
3. As x approaches infinity, the fractions containing x in their denominators approach 0.
4. $11/9$

Answer: 11/9

PRACTICE 6 problems

- 1** Find $\lim$ as x approaches infinity of $(7x^2 - 2x + 1)/(12x^2 + 6)$.

ANSWER _____

- 2** Find $\lim$ as x approaches negative infinity of $(13x^6 + 1)/(10x^6 - 7x)$.

ANSWER _____

- 3** Find $\lim$ as x approaches infinity of $(14x + 2)/(15x^2 - 1)$.

ANSWER _____

- 4** Find $\lim$ as x approaches negative infinity of $(16x^6 + x)/(17x^2 + 2)$.

ANSWER _____

- 5** Find $\lim$ as x approaches infinity of $(-18x^2 + 7)/(19x^2 - 10x)$.

ANSWER _____

- 6** Find $\lim$ as x approaches negative infinity of $(20x^{14} + 1)/(21x^{12} - 13)$.

ANSWER _____

APPLY IT Show your work

- 1** An online creator models the fraction of viewers who watch a full video after x editing revisions by $(31x^2 + 6x)/(37x^2 + 2)$. As x becomes very large, what fraction does the model approach?

WORK SPACE

ANSWER _____

- 2** A sneaker resale app models the fraction of listed items that sell within a week after x days of data collection by $(29x^2 + 6)/(34x^2 + 7)$. As x becomes very large, what fraction does the model approach?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find $\lim$ as x approaches negative infinity of $\sqrt{(36x^2 + 2x)/(12x - 1)}$. In one sentence, explain why the limit has a negative sign.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\lim$ as x approaches infinity of $(7x^2 - 2x + 1)/(12x^2 + 6)$.	7/12
2	Find $\lim$ as x approaches negative infinity of $(13x^6 + 1)/(10x^6 - 7x)$.	13/10
3	Find $\lim$ as x approaches infinity of $(14x + 2)/(15x^2 - 1)$.	0
4	Find $\lim$ as x approaches negative infinity of $(16x^6 + x)/(17x^2 + 2)$.	+infinity
5	Find $\lim$ as x approaches infinity of $(-18x^2 + 7)/(19x^2 - 10x)$.	-18/19
6	Find $\lim$ as x approaches negative infinity of $(20x^{14} + 1)/(21x^{12} - 13)$.	+infinity

PART 2 · APPLY IT

1. **31/37** Both polynomials have degree 2, so use the ratio of the leading coefficients, 31/37.
2. **29/34** Both polynomials have degree 2, so use the ratio of the leading coefficients, 29/34.

CHALLENGE

The limit is -1/2 because $\sqrt{36x^2 + 2x}$ behaves like $6|x|$, which equals $-6x$ for negative x , while $12x - 1$ behaves like $12x$.

Accept +infinity or infinity for unbounded positive limits and -infinity for unbounded negative limits. For the challenge, accept an explanation that uses $|x| = -x$ as x approaches negative infinity.