LESSON MAKER PRO

Infinity End Behavior

Find what rational expressions approach as x grows without bound.

HIGH SCHOOL · MA.912.C.1.5

MA.912.C.1.5 "Find limits at infinity."

NAME _____ DATE _____

FOCUS ON LEADING TERMS

To find a limit at infinity for a rational expression, compare the terms with the highest powers of x. If the degrees match, use the ratio of the leading coefficients. If the numerator has a smaller degree, the limit is 0, and if it has a larger degree, use the leading terms to determine whether the expression grows positively or negatively.

WORKED EXAMPLE

Find lim as x approaches infinity of $(11x^4 - 3x + 8)/(9x^4 + 5)$.

1. Divide the numerator and denominator by x^4.
2. $(11 - 3/x^3 + 8/x^4)/(9 + 5/x^4)$
3. As x approaches infinity, the fractions containing x in their denominators approach 0.
4. 11/9

Answer: 11/9

PRACTICE 6 problems

1 Find lim as x approaches infinity of $(7x^2 - 2x + 1)/(12x^2 + 6)$.

ANSWER _____

2 Find lim as x approaches negative infinity of $(13x^6 + 1)/(10x^6 - 7x)$.

ANSWER _____

3 Find lim as x approaches infinity of $(14x + 2)/(15x^2 - 1)$.

ANSWER _____

4 Find lim as x approaches negative infinity of $(16x^6 + x)/(17x^2 + 2)$.

ANSWER _____

5 Find lim as x approaches infinity of $(-18x^2 + 7)/(19x^2 - 10x)$.

ANSWER _____

6 Find lim as x approaches negative infinity of $(20x^{14} + 1)/(21x^{12} - 13)$.

ANSWER _____

APPLY IT Show your work

1 An online creator models the fraction of viewers who watch a full video after x editing revisions by $(31x^2 + 6x)/(37x^2 + 2)$. As x becomes very large, what fraction does the model approach?

WORK SPACE

ANSWER _____

2 A sneaker resale app models the fraction of listed items that sell within a week after x days of data collection by $(29x^2 + 6)/(34x^2 + 7)$. As x becomes very large, what fraction does the model approach?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find lim as x approaches negative infinity of $\text{sqrt}(36x^2 + 2x)/(12x - 1)$. In one sentence, explain why the limit has a negative sign.

WORK SPACE AND ANSWER

ANSWER KEY

Infinity End Behavior

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find lim as x approaches infinity of $(7x^2 - 2x + 1)/(12x^2 + 6)$.	**7/12**
2	Find lim as x approaches negative infinity of $(13x^6 + 1)/(10x^6 - 7x)$.	**13/10**
3	Find lim as x approaches infinity of $(14x + 2)/(15x^2 - 1)$.	**0**
4	Find lim as x approaches negative infinity of $(16x^6 + x)/(17x^2 + 2)$.	**+infinity**
5	Find lim as x approaches infinity of $(-18x^2 + 7)/(19x^2 - 10x)$.	**-18/19**
6	Find lim as x approaches negative infinity of $(20x^{14} + 1)/(21x^{12} - 13)$.	**+infinity**

PART 2 · APPLY IT

1. **31/37** Both polynomials have degree 2, so use the ratio of the leading coefficients, 31/37.
2. **29/34** Both polynomials have degree 2, so use the ratio of the leading coefficients, 29/34.

CHALLENGE

The limit is -1/2 because sqrt($36x^2 + 2x$) behaves like 6|x|, which equals -6x for negative x, while 12x - 1 behaves like 12x.

Accept +infinity or infinity for unbounded positive limits and -infinity for unbounded negative limits. For the challenge, accept an explanation that uses |x| = -x as x approaches negative infinity.