LESSON MAKER PRO

Infinity Ahead

Use limits to reveal vertical and horizontal asymptotes.

HIGH SCHOOL · MA.912.C.1.6

MA.912.C.1.6 "Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior."

NAME _____ DATE _____

WHEN LIMITS GROW

When a denominator approaches zero while the numerator stays nonzero, the quotient may increase or decrease without bound. Check the sign of the numerator and denominator from each side to decide whether the limit is +infinity or -infinity. As x approaches positive or negative infinity, compare the leading terms to describe end behavior and horizontal asymptotes.

WORKED EXAMPLE

Evaluate lim x->11+ (x+7)/(x-11).

1. Near x = 11 from the right, x - 11 is a tiny positive number.
2. x + 7 approaches 18, which is positive.
3. A positive number divided by a tiny positive number grows without bound.

Answer: lim x->11+ (x+7)/(x-11) = +infinity

PRACTICE 6 problems

1 Evaluate lim x->2+ 5/(x-2).

ANSWER _____

2 Evaluate lim x->-1- 6/(x+1).

ANSWER _____

3 Evaluate lim x->3 (x+2)/(x-3). State whether the two-sided limit exists.

ANSWER _____

4 For f(x) = (4x-1)/(x+5), identify the vertical asymptote and evaluate both one-sided limits there.

ANSWER _____

5 Evaluate lim x->infinity (2x^2-x+8)/(x^2+6), then state the horizontal asymptote.

ANSWER _____

6 Evaluate lim x->-infinity (x^3-2)/(2x^2+1).

ANSWER _____

APPLY IT Show your work

1 A game server models lag, in milliseconds, by L(p) = 90/(120-p), where p is the number of players online and p is less than 120. What happens to lag as p approaches 120 from below? What does this tell you about the graph?

WORK SPACE

ANSWER _____

2 A club orders hoodies. The average cost per hoodie is C(n) = (20n+100)/n dollars, where n is the number of hoodies ordered. Find lim n->infinity C(n) and explain what it means.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For g(x) = (2x+1)/(x^2-9), identify all vertical and horizontal asymptotes. Then give the one-sided limits at each vertical asymptote and explain how the signs of the factors determine your answers.

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_____

ANSWER KEY

Infinity Ahead

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate lim x->2+ 5/(x-2).	**+infinity**
2	Evaluate lim x->-1- 6/(x+1).	**-infinity**
3	Evaluate lim x->3 (x+2)/(x-3). State whether the two-sided limit exists.	**The limit does not exist. lim x->3- (x+2)/(x-3) = -infinity and lim x->3+ (x+2)/(x-3) = +infinity.**
4	For f(x) = (4x-1)/(x+5), identify the vertical asymptote and evaluate both one-sided limits there.	**Vertical asymptote: x = -5. lim x->-5- f(x) = +infinity and lim x->-5+ f(x) = -infinity.**
5	Evaluate lim x->infinity (2x^2-x+8)/(x^2+6), then state the horizontal asymptote.	**The limit is 2, and the horizontal asymptote is y = 2.**
6	Evaluate lim x->-infinity (x^3-2)/(2x^2+1).	**-infinity**

PART 2 · APPLY IT

1. **As p approaches 120 from below, L(p) approaches +infinity. The lag grows without bound, and p = 120 is a vertical asymptote.** As 120-p approaches zero through positive values, a positive numerator divided by a tiny positive denominator approaches +infinity.
2. **lim n->infinity C(n) = 20. The average cost approaches $20 per hoodie as the order size becomes very large, so y = 20 is a horizontal asymptote.** Rewrite C(n) as 20 + 100/n; as n grows without bound, 100/n approaches zero.

CHALLENGE

Vertical asymptotes: x = -3 and x = 3. lim x->-3- g(x) = -infinity, lim x->-3+ g(x) = +infinity, lim x->3- g(x) = -infinity, and lim x->3+ g(x) = +infinity. The horizontal asymptote is y = 0. The signs of (x-3) and (x+3) determine the denominator's sign on each side, while 2x+1 has a fixed sign near each asymptote.

Accept +∞ and -∞ as equivalent notation for +infinity and -infinity. For two-sided limits with opposite infinite one-sided behavior, accept "does not exist" or "DNE" only when the one-sided limits are correctly stated.