LESSON MAKER PRO

Limit Breakers

Use algebra and bounds to uncover hidden limits.

HIGH SCHOOL · MA.912.C.1.7

MA.912.C.1.7 "Find special limits by using the Squeeze Theorem or algebraic manipulation."

NAME _____ DATE _____

UNLOCKING A LIMIT

When direct substitution gives 0/0, factor the expression or multiply by a conjugate to remove the common factor. Then substitute the approaching value into the simplified expression. If a function is trapped between two expressions that both approach the same number, its limit is that number.

WORKED EXAMPLE

Find lim as x approaches 11 of $(x^2 - 121)/(x - 11)$.

1. Factor: $x^2 - 121 = (x - 11)(x + 11)$.
2. Cancel $x - 11$ for x not equal to 11.
3. Substitute 11: $11 + 11 = 22$.

Answer: 22

PRACTICE 6 problems

1 Find lim as x approaches 4 of $(x^2 - 16)/(x - 4)$.

ANSWER _____

2 Find lim as x approaches -2 of $(x^2 + 5x + 6)/(x + 2)$.

ANSWER _____

3 Find lim as x approaches 5 of $(\sqrt{x + 4} - 3)/(x - 5)$.

ANSWER _____

4 Find lim as x approaches 0 of $x^2 \sin(1/x)$.

ANSWER _____

5 Find lim as x approaches 1 of $(x^3 - 1)/(x - 1)$.

ANSWER _____

6 Find lim as x approaches -3 of $(x^2 + 2x - 3)/(x + 3)$.

ANSWER _____

APPLY IT Show your work

1 An esports club models a tournament stream's total views by $V(t) = t^2 + 9$, where V is in thousands of views and t is in hours. Find the instantaneous rate of change in views at $t = 6$ by evaluating lim as t approaches 6 of $(V(t) - V(6))/(t - 6)$.

WORK SPACE

ANSWER _____

2 A content creator models followers by $F(t) = \text{sqrt}(t + 16)$, where F is in thousands of followers and t is in days. Find the instantaneous rate of change at $t = 9$ by evaluating lim as t approaches 9 of $(F(t) - F(9))/(t - 9)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find lim as x approaches 2 of $(\text{sqrt}(x + 7) - 3)/(x^2 - 4)$. Then explain why cancellation is valid even though the original expression is undefined at $x = 2$.

WORK SPACE AND ANSWER

ANSWER KEY # Limit Breakers

HIGH SCHOOL · MA.912.C.1.7

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find lim as x approaches 4 of $(x^2 - 16)/(x - 4)$.	**8**
2	Find lim as x approaches -2 of $(x^2 + 5x + 6)/(x + 2)$.	**1**
3	Find lim as x approaches 5 of $(\sqrt{x + 4} - 3)/(x - 5)$.	**1/6**
4	Find lim as x approaches 0 of $x^2 \sin(1/x)$.	**0**
5	Find lim as x approaches 1 of $(x^3 - 1)/(x - 1)$.	**3**
6	Find lim as x approaches -3 of $(x^2 + 2x - 3)/(x + 3)$.	**-4**

PART 2 · APPLY IT

1. **12 thousand views per hour** $V(t) - V(6) = t^2 - 36 = (t - 6)(t + 6)$. After canceling $t - 6$, substitute 6 to get 12.
2. **1/10 thousand followers per day** Rationalize the numerator, then cancel $t - 9$. The simplified expression is $1/(\sqrt{t + 16} + 5)$, which approaches 1/10.

CHALLENGE

1/24. Cancellation is valid for x not equal to 2, and a limit depends on values near 2, not the value at 2.

Accept equivalent simplified forms and valid factorization, conjugate rationalization, or squeeze reasoning. For the word problems, accept equivalent rates with the correct units.