

Limit Detective

Use derivatives to unlock indeterminate limits.

HIGH SCHOOL • MA.912.C.1.8

MA.912.C.1.8 "Find limits of indeterminate forms using L'Hôpital's Rule."

NAME _____

DATE _____

WHEN TO USE THE RULE

Use L'Hôpital's Rule only when direct substitution gives $0/0$ or infinity/infinity. Differentiate the numerator and denominator separately, then find the new limit. Repeat the process if the new limit is still indeterminate.

WORKED EXAMPLE

Find $\lim_{x \rightarrow 0} \sin(7x)/x$.

1. Substitute $x = 0$: $\sin(7 \cdot 0)/0 = 0/0$.
2. Differentiate: the numerator becomes $7\cos(7x)$, and the denominator becomes 1.
3. Evaluate the new limit at $x = 0$: $7\cos(0)/1 = 7$.

Answer: 7

PRACTICE 6 problems

1 Find $\lim_{x \rightarrow 4} (x^2 - 16)/(x - 4)$.

ANSWER _____

2 Find $\lim_{x \rightarrow 0} \sin(x)/x$.

ANSWER _____

3 Find $\lim_{x \rightarrow 0} (e^x - 1)/x$.

ANSWER _____

4 Find $\lim_{x \rightarrow 1} \ln(x)/(x - 1)$.

ANSWER _____

5 Find $\lim_{x \rightarrow 0} (1 - \cos(x))/x^2$.

ANSWER _____

6 Find $\lim_{x \rightarrow \infty} (6x^2 + x)/(x^2 + 8)$.

ANSWER _____

APPLY IT Show your work

- 1** A student uses a social-media analytics model in which the average increase in views per unit of a small promotion level t is $\ln(1 + t)/t$. What value does the model approach as t approaches 0?

WORK SPACE

ANSWER _____

- 2** An esports team compares two practice-data scores. Its model for the ratio after t training sessions is $(8t^2 - 6t)/(2t^2 + t)$. What value does the ratio approach as t becomes very large?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find $\lim_{x \rightarrow 0} (\tan(x) - x)/x^3$. Show why L'Hôpital's Rule must be used more than once.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\lim_{x \rightarrow 4} \frac{(x^2 - 16)}{(x - 4)}$.	8
2	Find $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$.	1
3	Find $\lim_{x \rightarrow 0} \frac{(e^x - 1)}{x}$.	1
4	Find $\lim_{x \rightarrow 1} \frac{\ln(x)}{(x - 1)}$.	1
5	Find $\lim_{x \rightarrow 0} \frac{(1 - \cos(x))}{x^2}$.	1/2
6	Find $\lim_{x \rightarrow \infty} \frac{(6x^2 + x)}{(x^2 + 8)}$.	6

PART 2 · APPLY IT

- 1** Direct substitution gives $0/0$. Differentiate to get $[1/(1 + t)]/1$, then evaluate at $t = 0$.
- 4** The expression has the infinity/infinity form. Differentiate to get $(16t - 6)/(4t + 1)$, whose limit as t approaches infinity is 4.

CHALLENGE

1/3

Accept equivalent forms such as 0.5 for $1/2$. Students should show an initial $0/0$ or infinity/infinity form before applying L'Hôpital's Rule, and the challenge requires three applications.