LESSON MAKER PRO

Limit Detective

Use derivatives to unlock indeterminate limits.

HIGH SCHOOL · MA.912.C.1.8

MA.912.C.1.8 "Find limits of indeterminate forms using L'Hôpital's Rule."

NAME _____ DATE _____

WHEN TO USE THE RULE

Use L'Hôpital's Rule only when direct substitution gives 0/0 or infinity/infinity. Differentiate the numerator and denominator separately, then find the new limit. Repeat the process if the new limit is still indeterminate.

WORKED EXAMPLE

Find lim x->0 of sin(7x)/x.

1. Substitute x = 0: sin(7·0)/0 = 0/0.
2. Differentiate: the numerator becomes 7cos(7x), and the denominator becomes 1.
3. Evaluate the new limit at x = 0: 7cos(0)/1 = 7.

Answer: 7

PRACTICE 6 problems

1 Find lim x->4 of $(x^2 - 16)/(x - 4)$.

ANSWER _____

2 Find lim x->0 of sin(x)/x.

ANSWER _____

3 Find lim x->0 of $(e^x - 1)/x$.

ANSWER _____

4 Find lim x->1 of ln(x)/(x - 1).

ANSWER _____

5 Find lim x->0 of $(1 - \cos(x))/x^2$.

ANSWER _____

6 Find lim x->infinity of $(6x^2 + x)/(x^2 + 8)$.

ANSWER _____

APPLY IT Show your work

1 A student uses a social-media analytics model in which the average increase in views per unit of a small promotion level t is ln(1 + t)/t. What value does the model approach as t approaches 0?

WORK SPACE

ANSWER _____

2 An esports team compares two practice-data scores. Its model for the ratio after t training sessions is $(8t^2 - 6t)/(2t^2 + t)$. What value does the ratio approach as t becomes very large?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find lim x->0 of $(\tan(x) - x)/x^3$. Show why L'Hôpital's Rule must be used more than once.

WORK SPACE AND ANSWER

Limit Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find lim x->4 of $(x^2 - 16)/(x - 4)$.	**8**
2	Find lim x->0 of $\sin(x)/x$.	**1**
3	Find lim x->0 of $(e^x - 1)/x$.	**1**
4	Find lim x->1 of $\ln(x)/(x - 1)$.	**1**
5	Find lim x->0 of $(1 - \cos(x))/x^2$.	**1/2**
6	Find lim x->infinity of $(6x^2 + x)/(x^2 + 8)$.	**6**

PART 2 · APPLY IT

1. **1** Direct substitution gives 0/0. Differentiate to get [1/(1 + t)]/1, then evaluate at t = 0.
2. **4** The expression has the infinity/infinity form. Differentiate to get (16t - 6)/(4t + 1), whose limit as t approaches infinity is 4.

CHALLENGE

1/3

Accept equivalent forms such as 0.5 for 1/2. Students should show an initial 0/0 or infinity/infinity form before applying L'Hôpital's Rule, and the challenge requires three applications.