

Limit Checkpoint

Use limits to decide whether a function is continuous.

HIGH SCHOOL • MA.912.C.1.9

MA.912.C.1.9 "Define continuity in terms of limits."

NAME _____

DATE _____

CONTINUITY TEST

A function is continuous at $x=a$ when $\lim_{x \rightarrow a} f(x)$ exists, $f(a)$ exists, and $\lim_{x \rightarrow a} f(x) = f(a)$. Check the left-hand and right-hand limits first. If they are different, the limit does not exist, so the function is not continuous.

WORKED EXAMPLE

Decide whether $f(x) = 7x - 11$ is continuous at $x = 8$.

1. Find the limit: $\lim_{x \rightarrow 8} (7x - 11) = 7(8) - 11$.
2. Compute the value: $56 - 11 = 45$.
3. Find $f(8) = 45$, which matches the limit.

Answer: f is continuous at $x = 8$.

PRACTICE 6 problems

- 1** For $f(x) = x^2 + 4$, find $\lim_{x \rightarrow 2} f(x)$, find $f(2)$, and decide whether f is continuous at $x = 2$.

ANSWER _____

- 2** For $g(x) = x + 1$ if $x < 1$, and $g(x) = 2x$ if $x \geq 1$, find the left-hand limit, right-hand limit, and $g(1)$. Is g continuous at $x = 1$?

ANSWER _____

- 3** For $h(x) = (x^2 - 25)/(x - 5)$, decide whether h is continuous at $x = 5$.

ANSWER _____

- 4** For $p(x) = x + 6$ if $x < 0$, and $p(x) = 3x + 1$ if $x \geq 0$, decide whether p is continuous at $x = 0$.

ANSWER _____

- 5** For $q(x) = (x^2 + x - 20)/(x - 4)$, find $\lim_{x \rightarrow 4} q(x)$ and decide whether q is continuous at $x = 4$.

ANSWER _____

- 6** Find k so that $r(x) = x + 2$ if $x < 4$, and $r(x) = k$ if $x \geq 4$, is continuous at $x = 4$.

ANSWER _____

APPLY IT Show your work

- 1** At a school concert, a light's brightness $B(t)$ is modeled by $B(t)=t+4$ if $t<2$, and $B(t)=2t+2$ if $t\geq 2$, where t is time in minutes. Is the brightness model continuous at $t=2$?

WORK SPACE

ANSWER _____

- 2** In an online game, a bonus $B(r)$ is modeled by $B(r)=r+1$ if $r<5$, and $B(r)=2r$ if $r\geq 5$, where r is a player's performance rating. Is the bonus model continuous at $r=5$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

The function $s(x)=(x^2-1)/(x-1)$ if $x\neq 1$, and $s(x)=a$ if $x=1$. Find a so that s is continuous at $x=1$. Explain your reasoning.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x)=x^2+4$, find $\lim_{x \rightarrow 2} f(x)$, find $f(2)$, and decide whether f is continuous at $x=2$.	$\lim_{x \rightarrow 2} f(x)=8$ and $f(2)=8$, so f is continuous at $x=2$.
2	For $g(x)=x+1$ if $x<1$, and $g(x)=2x$ if $x \geq 1$, find the left-hand limit, right-hand limit, and $g(1)$. Is g continuous at $x=1$?	The left-hand limit is 2, the right-hand limit is 2, and $g(1)=2$. g is continuous at $x=1$.
3	For $h(x)=(x^2-25)/(x-5)$, decide whether h is continuous at $x=5$.	$\lim_{x \rightarrow 5} h(x)=10$, but $h(5)$ is undefined. h is not continuous at $x=5$.
4	For $p(x)=x+6$ if $x<0$, and $p(x)=3x+1$ if $x \geq 0$, decide whether p is continuous at $x=0$.	The left-hand limit is 6 and the right-hand limit is 1, so the limit does not exist. p is not continuous at $x=0$.
5	For $q(x)=(x^2+x-20)/(x-4)$, find $\lim_{x \rightarrow 4} q(x)$ and decide whether q is continuous at $x=4$.	$\lim_{x \rightarrow 4} q(x)=9$, but $q(4)$ is undefined. q is not continuous at $x=4$.
6	Find k so that $r(x)=x+2$ if $x<4$, and $r(x)=k$ if $x \geq 4$, is continuous at $x=4$.	$k=6$.

PART 2 · APPLY IT

- Yes, the model is continuous at $t=2$.** The left-hand limit is 6, the right-hand limit is 6, and $B(2)=6$. Since the limit exists and equals the function value, the model is continuous.
- No, the model is not continuous at $r=5$.** The left-hand limit is 6, while the right-hand limit and $B(5)$ are 10. Because the one-sided limits are different, the limit does not exist.

CHALLENGE

$a=2$. For $x \neq 1$, $(x^2-1)/(x-1)=x+1$, so $\lim_{x \rightarrow 1} s(x)=2$. Setting $s(1)=2$ makes the function continuous.

Accept equivalent factorization and limit work. For continuity, students must show that the limit exists, the function value exists, and the two values are equal.