LESSON MAKER PRO

Limit Checkpoint

Use limits to decide whether a function is continuous.

HIGH SCHOOL · MA.912.C.1.9

MA.912.C.1.9 "Define continuity in terms of limits."

NAME _____ DATE _____

CONTINUITY TEST

A function is continuous at x=a when lim x->a f(x) exists, f(a) exists, and lim x->a f(x)=f(a). Check the left-hand and right-hand limits first. If they are different, the limit does not exist, so the function is not continuous.

WORKED EXAMPLE

Decide whether f(x)=7x-11 is continuous at x=8.

1. Find the limit: lim x->8 (7x-11)=7(8)-11.
2. Compute the value: 56-11=45.
3. Find f(8)=45, which matches the limit.

Answer: f is continuous at x=8.

PRACTICE 6 problems

1 For $f(x)=x^2+4$, find lim x->2 f(x), find f(2), and decide whether f is continuous at x=2.

ANSWER _____

2 For g(x)=x+1 if x<1, and g(x)=2x if x>=1, find the left-hand limit, right-hand limit, and g(1). Is g continuous at x=1?

ANSWER _____

3 For $h(x)=(x^2-25)/(x-5)$, decide whether h is continuous at x=5.

ANSWER _____

4 For p(x)=x+6 if x<0, and p(x)=3x+1 if x>=0, decide whether p is continuous at x=0.

ANSWER _____

5 For $q(x)=(x^2+x-20)/(x-4)$, find lim x->4 q(x) and decide whether q is continuous at x=4.

ANSWER _____

6 Find k so that r(x)=x+2 if x<4, and r(x)=k if x>=4, is continuous at x=4.

ANSWER _____

APPLY IT Show your work

1 At a school concert, a light's brightness B(t) is modeled by B(t)=t+4 if t<2, and B(t)=2t+2 if t>=2, where t is time in minutes. Is the brightness model continuous at t=2?

WORK SPACE

ANSWER _____

2 In an online game, a bonus B(r) is modeled by B(r)=r+1 if r<5, and B(r)=2r if r>=5, where r is a player's performance rating. Is the bonus model continuous at r=5?

WORK SPACE

ANSWER _____

CHALLENGE One step up

The function $s(x)=(x^2-1)/(x-1)$ if x!=1, and s(x)=a if x=1. Find a so that s is continuous at x=1. Explain your reasoning.

WORK SPACE AND ANSWER

ANSWER KEY # Limit Checkpoint

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x)=x^2+4$, find lim x->2 f(x), find f(2), and decide whether f is continuous at x=2.	**lim x->2 f(x)=8 and f(2)=8, so f is continuous at x=2.**
2	For g(x)=x+1 if x<1, and g(x)=2x if x>=1, find the left-hand limit, right-hand limit, and g(1). Is g continuous at x=1?	**The left-hand limit is 2, the right-hand limit is 2, and g(1)=2. g is continuous at x=1.**
3	For $h(x)=(x^2-25)/(x-5)$, decide whether h is continuous at x=5.	**lim x->5 h(x)=10, but h(5) is undefined. h is not continuous at x=5.**
4	For p(x)=x+6 if x<0, and p(x)=3x+1 if x>=0, decide whether p is continuous at x=0.	**The left-hand limit is 6 and the right-hand limit is 1, so the limit does not exist. p is not continuous at x=0.**
5	For $q(x)=(x^2+x-20)/(x-4)$, find lim x->4 q(x) and decide whether q is continuous at x=4.	**lim x->4 q(x)=9, but q(4) is undefined. q is not continuous at x=4.**
6	Find k so that r(x)=x+2 if x<4, and r(x)=k if x>=4, is continuous at x=4.	**k=6.**

PART 2 · APPLY IT

1. **Yes, the model is continuous at t=2.** The left-hand limit is 6, the right-hand limit is 6, and B(2)=6. Since the limit exists and equals the function value, the model is continuous.
2. **No, the model is not continuous at r=5.** The left-hand limit is 6, while the right-hand limit and B(5) are 10. Because the one-sided limits are different, the limit does not exist.

CHALLENGE

a=2. For x!=1, $(x^2-1)/(x-1)=x+1$, so lim x->1 s(x)=2. Setting s(1)=2 makes the function continuous.

Accept equivalent factorization and limit work. For continuity, students must show that the limit exists, the function value exists, and the two values are equal.