

Slope in a Snap

Use limits, slopes, and rates of change to find derivatives.

HIGH SCHOOL · MA.912.C.2.1

MA.912.C.2.1 "State, understand and apply the definition of derivative. Apply and interpret derivatives geometrically and numerically."

NAME _____

DATE _____

DERIVATIVE BASICS

The derivative $f'(a)$ is the limit of the average rate of change as the change in x gets closer to zero: $f'(a) = \lim_{h \rightarrow 0} [f(a+h) - f(a)]/h$. It gives the slope of the tangent line at $x=a$ and the instantaneous rate of change at that input.

WORKED EXAMPLE

Use the definition of derivative to find $f'(7)$ for $f(x) = x^2$.

$$1. f'(7) = \lim_{h \rightarrow 0} [f(7+h) - f(7)]/h$$

$$2. = \lim_{h \rightarrow 0} [(7+h)^2 - 49]/h$$

$$3. = \lim_{h \rightarrow 0} [h(14+h)]/h$$

$$4. = \lim_{h \rightarrow 0} (14+h) = 14$$

Answer: $f'(7) = 14$

PRACTICE 6 problems

- 1** Use the limit definition to find $f'(1)$ for $f(x) = x^3$.

ANSWER _____

- 2** The tangent line to a curve at $x=1$ is $y=9x-8$. What is $f'(1)$?

ANSWER _____

- 3** A function has $f(1/3)=11$ and $f(5/3)=3$. Use the symmetric difference quotient to estimate $f'(1)$.

ANSWER _____

- 4** Find $g'(8)$ for $g(x) = 1/x$.

ANSWER _____

- 5** Find $h'(x)$ for $h(x) = 5x-11$.

ANSWER _____

- 6** A curve has a tangent slope of 12 at $x=-3$. State $f'(-3)$.

ANSWER _____

APPLY IT Show your work

- 1** A streaming creator models the total number of new followers, in hundreds, after t hours by $P(t)=t^3+4t$. Find the instantaneous rate at $t=3$. Interpret the result.

WORK SPACE

ANSWER _____

- 2** In a racing video game, a car's signed position is modeled by $s(t)=t^4-8t$, where t is in seconds. Find the car's instantaneous velocity at $t=1$ and interpret the sign.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Use the limit definition to find $f'(x)$ for $f(x)=x^4-5x$. Then find the equation of the tangent line when $x=1$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Use the limit definition to find $f'(1)$ for $f(x)=x^3$.	3
2	The tangent line to a curve at $x=1$ is $y=9x-8$. What is $f'(1)$?	9
3	A function has $f(1/3)=11$ and $f(5/3)=3$. Use the symmetric difference quotient to estimate $f'(1)$.	-6
4	Find $g'(8)$ for $g(x)=1/x$.	-1/64
5	Find $h'(x)$ for $h(x)=5x-11$.	5
6	A curve has a tangent slope of 12 at $x=-3$. State $f'(-3)$.	12

PART 2 • APPLY IT

- 31 hundreds of followers per hour, or 3,100 followers per hour** Differentiate to get $P'(t)=3t^2+4$. Then $P'(3)=31$, and each unit represents one hundred followers per hour.
- 4 position units per second; the car is moving in the negative direction** Differentiate to get $s'(t)=4t^3-8$. Evaluating at $t=1$ gives -4, so the negative sign shows motion in the negative direction.

CHALLENGE

$f'(x)=4x^3-5$; tangent line: $y=-x-3$

Accept equivalent derivative forms and the equivalent tangent-line form $y+4=-(x-1)$. For the numerical estimate, accept -6 when students correctly use the centered secant slope.