LESSON MAKER PRO

Slope Match Mission

Find an instant that matches an interval's average rate of change.

HIGH SCHOOL · MA.912.C.2.10

MA.912.C.2.10 "Apply the Mean Value Theorem."

NAME _____ DATE _____

THE MEAN VALUE LINK

The Mean Value Theorem applies when f is continuous on [a,b] and differentiable on (a,b). It guarantees at least one number c in (a,b) where f'(c) = (f(b)-f(a))/(b-a). First check the conditions, then find the average rate of change and solve for c.

WORKED EXAMPLE

Use the Mean Value Theorem to find c for f(x)=x^2+13 on [5,11].

1. f is a polynomial, so it is continuous on [5,11] and differentiable on (5,11).
2. (f(11)-f(5))/(11-5) = (134-38)/6 = 16.
3. f'(x)=2x, so 2c=16.
4. c=8, and 8 is in (5,11).

Answer: c=8

PRACTICE 6 problems

1 For f(x)=3x-2 on [0,4], find all values of c guaranteed by the Mean Value Theorem.

ANSWER _____

2 Use the Mean Value Theorem to find c for f(x)=x^2-1 on [-2,2].

ANSWER _____

3 Use the Mean Value Theorem to find c for f(x)=x^3 on [1,2].

ANSWER _____

4 Use the Mean Value Theorem to find c for f(x)=1/x on [1,4].

ANSWER _____

5 Does the Mean Value Theorem apply to f(x)=|x| on [-3,1]? Explain briefly.

ANSWER _____

6 Use the Mean Value Theorem to find c for f(x)=sin x on [0,pi].

ANSWER _____

APPLY IT Show your work

1 During a 4-hour bike ride, Jordan's distance from the start, in miles, is modeled by s(t)=t^2+2t for 0<=t<=4. Use the Mean Value Theorem to find a time when Jordan's instantaneous velocity equals the average velocity for the entire ride.

WORK SPACE

ANSWER _____

2 A game download has q(t)=t^2+1 gigabytes downloaded t hours after it starts, for 1<=t<=3. Find a time when the instantaneous download rate equals the average download rate over that interval.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For f(x)=x^3-3x on [-2,2], use the Mean Value Theorem to find all values of c. Explain why each value works.

WORK SPACE AND ANSWER

ANSWER KEY # Slope Match Mission

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x)=3x-2 on [0,4], find all values of c guaranteed by the Mean Value Theorem.	**Every c with 0<c<4.**
2	Use the Mean Value Theorem to find c for f(x)=x^2-1 on [-2,2].	**c=0**
3	Use the Mean Value Theorem to find c for f(x)=x^3 on [1,2].	**c=sqrt(7/3)**
4	Use the Mean Value Theorem to find c for f(x)=1/x on [1,4].	**c=2**
5	Does the Mean Value Theorem apply to f(x)=|x| on [-3,1]? Explain briefly.	**No. f is not differentiable at x=0, which is inside (-3,1).**
6	Use the Mean Value Theorem to find c for f(x)=sin x on [0,pi].	**c=pi/2**

PART 2 · APPLY IT

1. **t=2 hours** The average velocity is (s(4)-s(0))/(4-0)=6 miles per hour. Since s'(t)=2t+2, solve 2t+2=6 to get t=2.
2. **t=2 hours** The average download rate is (q(3)-q(1))/(3-1)=4 gigabytes per hour. Since q'(t)=2t, solve 2t=4 to get t=2.

CHALLENGE

f is a polynomial, so the theorem applies. The average rate of change is (f(2)-f(-2))/(2-(-2))=(2-(-2))/4=1. Since f'(x)=3x^2-3, solve 3c^2-3=1 to get c=+/-2/sqrt(3). Both values are in (-2,2).

Accept sqrt(7/3) or sqrt(21)/3. Accept +/-2/sqrt(3) or +/-2sqrt(3)/3 for the challenge, and accept any equivalent statement that every interior point works for the linear function.