LESSON MAKER PRO

Hidden Curve Slopes

Use implicit differentiation to find rates on connected curves.

HIGH SCHOOL · MA.912.C.2.5

MA.912.C.2.5 "Find the derivatives of implicitly defined functions."

NAME _____ DATE _____

TRACK THE HIDDEN RATE

When y depends on x, differentiate every term with respect to x. When you differentiate a power of y, use the chain rule, so the derivative includes dy/dx. Then solve the new equation for dy/dx.

WORKED EXAMPLE

Find dy/dx if $x^5 + y^5 = 32$.

1. Differentiate both sides: $5x^4 + 5y^4(dy/dx) = 0$.
2. Isolate the dy/dx term: $5y^4(dy/dx) = -5x^4$.
3. Divide by $5y^4$: $dy/dx = -x^4/y^4$.

Answer: $dy/dx = -x^4/y^4$

PRACTICE 6 problems

1 Find dy/dx if $x^2 + y^2 = 9$.

ANSWER _____

2 Find dy/dx if $x^3 + y^2 = 7$.

ANSWER _____

3 Find dy/dx if $xy = 8$.

ANSWER _____

4 Find dy/dx if $x^2 + xy + y^2 = 12$.

ANSWER _____

5 Find dy/dx if $x^2y + y^3 = 6$.

ANSWER _____

6 Find dy/dx if $x^2 + 2xy - y^2 = 1$.

ANSWER _____

APPLY IT Show your work

1 A robotics club programs a camera to follow the curve $x^2 + y^2 = 1681$, where x and y are coordinates in centimeters. At the point (9, 40), find the rate at which y changes with respect to x.

WORK SPACE

ANSWER _____

2 A music app links two sound settings, x and y, with $x^2 + xy + y^2 = 76$. At the setting (4, 6), find the rate at which y changes with respect to x.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For the curve $x^2 + y^2 = 36$, find d^2y/dx^2 in terms of y.

WORK SPACE AND ANSWER

ANSWER KEY

Hidden Curve Slopes

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find dy/dx if $x^2 + y^2 = 9$.	**$dy/dx = -x/y$**
2	Find dy/dx if $x^3 + y^2 = 7$.	**$dy/dx = -3x^2/(2y)$**
3	Find dy/dx if $xy = 8$.	**$dy/dx = -y/x$**
4	Find dy/dx if $x^2 + xy + y^2 = 12$.	**$dy/dx = -(2x + y)/(x + 2y)$**
5	Find dy/dx if $x^2y + y^3 = 6$.	**$dy/dx = -2xy/(x^2 + 3y^2)$**
6	Find dy/dx if $x^2 + 2xy - y^2 = 1$.	**$dy/dx = -(x + y)/(x - y)$**

PART 2 · APPLY IT

1. **$dy/dx = -9/40$** Differentiate to get $2x + 2y(dy/dx) = 0$, so $dy/dx = -x/y$. Substitute $x = 9$ and $y = 40$.
2. **$dy/dx = -7/8$** Implicit differentiation gives $dy/dx = -(2x + y)/(x + 2y)$. Substituting $x = 4$ and $y = 6$ gives $-14/16$, which simplifies to $-7/8$.

CHALLENGE

$d^2y/dx^2 = -36/y^3$

Accept equivalent algebraic forms, including unsimplified fractions with the same sign and value. For the challenge, accept $-(x^2 + y^2)/y^3$ because the original equation makes it equal to $-36/y^3$.