LESSON MAKER PRO

Inverse Rate Switch

Use reciprocal slopes to differentiate inverse functions.

HIGH SCHOOL · MA.912.C.2.6

MA.912.C.2.6 "Find derivatives of inverse functions."

NAME _____ DATE _____

THE BIG IDEA

For an inverse function, use $(f^{-1})'(x)=1/f'(f^{-1}(x))$. First find the original input that produces the inverse's input, then take the reciprocal of the derivative at that original input. The derivative in the denominator cannot be zero.

WORKED EXAMPLE

Given $f(12)=-11$ and $f'(12)=14$, find $(f^{-1})'(-11)$.

1. $f^{-1}(-11)=12$ because $f(12)=-11$.
2. Use $(f^{-1})'(-11)=1/f'(12)$.
3. Substitute $f'(12)=14$: $(f^{-1})'(-11)=1/14$.

Answer: 1/14

PRACTICE 6 problems

1 Given $f(2)=5$ and $f'(2)=3$, find $(f^{-1})'(5)$.

ANSWER _____

2 Given $g(-1)=6$ and $g'(-1)=-2$, find $(g^{-1})'(6)$.

ANSWER _____

3 Let $h(x)=x^2+2$ for $x>=0$. Find $(h^{-1})'(3)$.

ANSWER _____

4 Given $p^{-1}(8)=-5$ and $p'(-5)=10$, find $(p^{-1})'(8)$.

ANSWER _____

5 Let $q(x)=x^3+1$. Find $(q^{-1})'(2)$.

ANSWER _____

6 Use the table to find $(r^{-1})'(10)$. x: -3 1 5 r(x): -6 2 10 r'(x): 8 -5 6

ANSWER _____

APPLY IT Show your work

1 An esports team models its skill score as a differentiable one-to-one function of practice hours. After 5 hours, its score is 30 points, and at that time the score is increasing at 15 points per hour. The inverse model gives practice hours as a function of score. At a score of 30 points, find the rate at which practice hours change per point.

WORK SPACE

ANSWER _____

2 An indie musician's total followers is a differentiable one-to-one function of weeks after a song release. At 6 weeks, the musician has 500 followers, and the total is increasing at 25 followers per week. The inverse model gives weeks as a function of followers. At 500 followers, find the rate at which weeks change per follower.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A twice-differentiable one-to-one function f satisfies f(1)=2, f'(1)=3, and f''(1)=-5. Find $(f^{-1})''(2)$.

WORK SPACE AND ANSWER

ANSWER KEY

Inverse Rate Switch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given f(2)=5 and f'(2)=3, find $(f^{-1})'(5)$.	**1/3**
2	Given g(-1)=6 and g'(-1)=-2, find $(g^{-1})'(6)$.	**-1/2**
3	Let $h(x)=x^2+2$ for x>=0. Find $(h^{-1})'(3)$.	**1/2**
4	Given $p^{-1}(8)=-5$ and p'(-5)=10, find $(p^{-1})'(8)$.	**1/10**
5	Let $q(x)=x^3+1$. Find $(q^{-1})'(2)$.	**1/3**
6	Use the table to find $(r^{-1})'(10)$. x: -3 1 5 r(x): -6 2 10 r'(x): 8 -5 6	**1/6**

PART 2 · APPLY IT

1. **1/15 hour per point** The inverse derivative is the reciprocal of 15 points per hour. Therefore, practice hours change at 1/15 hour per point.
2. **1/25 week per follower** The inverse derivative is the reciprocal of 25 followers per week. Therefore, weeks change at 1/25 week per follower.

CHALLENGE

5/27

Accept equivalent fraction forms and reciprocal rates with correct units. For the challenge, use $(f^{-1})''(x)=-f''(f^{-1}(x))/(f'(f^{-1}(x)))^3$.