

Log Power Moves

Use logarithms to unlock complex derivatives.

HIGH SCHOOL • MA.912.C.2.8

MA.912.C.2.8 "Find derivatives using logarithmic differentiation."

NAME _____

DATE _____

TURN FACTORS INTO SUMS

When y has variable powers, products, or quotients, take $\ln$ of both sides, often using $\ln|y|$. Use log rules to change products into sums, quotients into differences, and powers into coefficients. Differentiate implicitly, then multiply by y to solve for y' .

WORKED EXAMPLE

Find dy/dx if $y = (x^7 + 11)^{13}/x^{17}$.

- $\ln|y| = 13\ln|x^7 + 11| - 17\ln|x|$
- $y'/y = 13(7x^6/(x^7 + 11)) - 17/x$
- $y' = y[91x^6/(x^7 + 11) - 17/x]$
- Replace y with the original expression.

Answer: $dy/dx = ((x^7 + 11)^{13}/x^{17}) [91x^6/(x^7 + 11) - 17/x]$

PRACTICE 6 problems

1 Find dy/dx for $x > 0$ if $y = x^x$.

ANSWER _____

2 Find dy/dx if $y = (x^3 + 1)^5/x^2$.

ANSWER _____

3 Find dy/dx if $y = (x - 4)^5(x + 2)^3$.

ANSWER _____

4 Find dy/dx if $y = (x^2 + 3)^4/(x - 1)^5$.

ANSWER _____

5 Find dy/dx if $y = (3x - 1)^5/(x^2 + 2)^4$.

ANSWER _____

6 Find dy/dx if $y = (x^4 + 1)^3(x^2 - 5)^2/x^5$.

ANSWER _____

APPLY IT Show your work

- 1** A student tracks the online share score of a video. For $x > 0$ hours after posting, the score is modeled by $y = (x+4)^5/x^3$. Find the instantaneous rate of change of the score, dy/dx .

WORK SPACE

ANSWER _____

- 2** A gaming club models its weekly engagement score by $y = (x^2+1)^5/(x+3)^4$, where x is the number of weeks since a tournament began. Find the instantaneous rate of change of the engagement score, dy/dx .

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find dy/dx if $y = (x^2+4)^5(x-3)^4/[x^3(2x+1)^5]$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find dy/dx for $x > 0$ if $y = x^x$.	$dy/dx = x^x(\ln x + 1)$
2	Find dy/dx if $y = (x^3 + 1)^5/x^2$.	$dy/dx = ((x^3 + 1)^5/x^2)$ $[15x^2/(x^3 + 1) - 2/x]$
3	Find dy/dx if $y = (x - 4)^5(x + 2)^3$.	$dy/dx = (x - 4)^5(x + 2)^3[5/(x - 4) + 3/(x + 2)]$
4	Find dy/dx if $y = (x^2 + 3)^4/(x - 1)^5$.	$dy/dx = ((x^2 + 3)^4/(x - 1)^5)$ $[8x/(x^2 + 3) - 5/(x - 1)]$
5	Find dy/dx if $y = (3x - 1)^5/(x^2 + 2)^4$.	$dy/dx = ((3x - 1)^5/(x^2 + 2)^4)[15/(3x - 1) - 8x/(x^2 + 2)]$
6	Find dy/dx if $y = (x^4 + 1)^3(x^2 - 5)^2/x^5$.	$dy/dx = ((x^4 + 1)^3(x^2 - 5)^2/x^5)$ $[12x^3/(x^4 + 1) + 4x/(x^2 - 5) - 5/x]$

PART 2 • APPLY IT

1. $dy/dx = ((x + 4)^5/x^3)[5/(x + 4) - 3/x]$ Take $\ln$ of both sides, then use the quotient and power rules for logarithms. Differentiate implicitly and multiply the result by the original y .
2. $dy/dx = ((x^2 + 1)^5/(x + 3)^4)[10x/(x^2 + 1) - 4/(x + 3)]$ Write $\ln|y|$ as a difference of logarithms. Differentiate each term, then multiply by the original function.

CHALLENGE

$$dy/dx = ((x^2 + 4)^5(x - 3)^4/[x^3(2x + 1)^5])[10x/(x^2 + 4) + 4/(x - 3) - 3/x - 10/(2x + 1)]$$

Accept equivalent derivatives that are expanded, factored, or written using the original function times its logarithmic derivative. Absolute value signs in intermediate logarithm work are acceptable and mathematically precise.