LESSON MAKER PRO

Log Power Moves

Use logarithms to differentiate products, quotients, and powers.

HIGH SCHOOL · MA.912.C.2.8

MA.912.C.2.8 "Find derivatives using logarithmic differentiation."

NAME _____ DATE _____

TURN PRODUCTS INTO SUMS

For a product, quotient, or power, take ln|y| so multiplication becomes addition, division becomes subtraction, and exponents become coefficients. Differentiate both sides, solve for y', and then multiply by the original function. If a factor can equal zero, simplify your final derivative so it can be checked at those values.

WORKED EXAMPLE

Find y' using logarithmic differentiation: $y = (x + 13)^6 / x^5$.

1. For x not equal to 0 or -13, take logs: $\ln|y| = 6\ln|x + 13| - 5\ln|x|$.
2. Differentiate: $y'/y = 6/(x + 13) - 5/x$.
3. Multiply by y and simplify: $y' = (x + 13)^5(x - 65) / x^6$.
4. The simplified expression is also valid at x = -13; the original function is undefined only at x = 0.

Answer: $y' = (x + 13)^5(x - 65) / x^6$

PRACTICE 6 problems

1 Find y' using logarithmic differentiation: $y = x^4(x + 2)^3$.

ANSWER _____

2 Find y' using logarithmic differentiation: $y = (x^2 + 1)^3 / x^2$.

ANSWER _____

3 Find y' using logarithmic differentiation: $y = (3x - 1)^4(x + 4)^2$.

ANSWER _____

4 Find y' using logarithmic differentiation: $y = ((x^2 + 3) / (x - 1))^2$.

ANSWER _____

5 Find y' using logarithmic differentiation: $y = (x^2 + 7)^{3/2}$.

ANSWER _____

6 Find y' using logarithmic differentiation: $y = (2x + 1)^3 / (x^2 + 3)^2$.

ANSWER _____

APPLY IT Show your work

1 A livestream platform models a creator's viewer-score growth by $V(x) = 120x^3 / (x + 2)^2$, where x is the number of hours after the stream begins. Find V'(1), the instantaneous change in viewer score after 1 hour.

WORK SPACE

ANSWER _____

2 A music app models a playlist's trend index by $I(x) = (x^2 + 4)^3 / (x + 2)^2$, where x is hours after a new song is added. Find I'(2), the instantaneous change in trend-index points after 2 hours.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Use logarithmic differentiation to find y' for $y = (x - 2)^4(x + 3)^3 / (x^2 + 1)^2$. Give a factored derivative that is valid for all real x, then explain in one sentence why it is valid at x = 2 and x = -3.

WORK SPACE AND ANSWER

ANSWER KEY # Log Power Moves

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find y' using logarithmic differentiation: $y = x^4(x + 2)^3$.	**$y' = x^3(x + 2)^2(7x + 8)$**
2	Find y' using logarithmic differentiation: $y = (x^2 + 1)^3 / x^2$.	**$y' = 2(x^2 + 1)^2(2x^2 - 1) / x^3$**
3	Find y' using logarithmic differentiation: $y = (3x - 1)^4(x + 4)^2$.	**$y' = 2(3x - 1)^3(x + 4)(9x + 23)$**
4	Find y' using logarithmic differentiation: $y = ((x^2 + 3) / (x - 1))^2$.	**$y' = 2(x^2 + 3)(x - 3)(x + 1) / (x - 1)^3$**
5	Find y' using logarithmic differentiation: $y = (x^2 + 7)^{3/2}$.	**$y' = 3x(x^2 + 7)^{1/2}$**
6	Find y' using logarithmic differentiation: $y = (2x + 1)^3 / (x^2 + 3)^2$.	**$y' = -2(2x + 1)^2(x^2 + 2x - 9) / (x^2 + 3)^3$**

PART 2 · APPLY IT

1. **V'(1) = 280/9 viewer-score points per hour** Use $V'/V = 3/x - 2/(x + 2)$. At $x = 1$, $V(1) = 40/3$ and the logarithmic derivative is 7/3, so $V'(1) = 280/9$.
2. **I'(2) = 32 trend-index points per hour** Use $I'/I = 6x/(x^2 + 4) - 2/(x + 2)$. At $x = 2$, $I(2) = 32$ and the logarithmic derivative equals 1, so $I'(2) = 32$.

CHALLENGE

$y' = (x - 2)^3(x + 3)^2(3x^3 + 2x^2 + 31x + 6) / (x^2 + 1)^3$. The denominator is never zero for real x, so this factored expression also gives the derivative at $x = 2$ and $x = -3$.

Accept algebraically equivalent derivatives. For products with factors that can be zero, accept a simplified derivative that remains defined at those zeros when the original function is differentiable there.