LESSON MAKER PRO

Smooth or Split?

Use continuity to decide when a derivative can exist.

HIGH SCHOOL · MA.912.C.2.9

MA.912.C.2.9 "Demonstrate and use the relationship between differentiability and continuity."

NAME _____ DATE _____

PRACTICE: THE LINK

If a function is differentiable at x=c, then it must be continuous at x=c. To test a piecewise function, first compare the left and right values at the joining point. If it is continuous, compare the left and right derivatives to decide whether it is differentiable.

WORKED EXAMPLE

Determine whether f is continuous and differentiable at x=11. $f(x)=x^2$ for $x<=11$, and $f(x)=22x-121$ for $x>11$.

1. $f(11)=11^2=121$.
2. The left-hand value is 121, and the right-hand value is 22(11)-121=121.
3. Both sides match f(11), so f is continuous at x=11.
4. The left derivative is 2(11)=22, and the right derivative is 22, so f is differentiable.

Answer: f is continuous and differentiable at x=11.

PRACTICE 6 problems

1 $p(x)=(x^2-1)/(x-1)$ for $x!=1$, and $p(1)=5$. Is p continuous and differentiable at x=1?

ANSWER _____

2 $q(x)=|x+2|$. Is q continuous and differentiable at x=-2?

ANSWER _____

3 $r(x)=x+1$ for $x<0$, and $r(x)=x^2+1$ for $x>=0$. Is r continuous and differentiable at x=0?

ANSWER _____

4 $s(x)=1/(x-3)$. Is s continuous and differentiable at x=3?

ANSWER _____

5 $t(x)=x^3-5x$. Is t continuous and differentiable at x=1?

ANSWER _____

6 Function m is differentiable at x=-1. What must be true about m at x=-1?

ANSWER _____

APPLY IT Show your work

1 An earbuds app models the battery level, in percentage points, after t hours of charging by B(t)=20t+30 for t<=2 and B(t)=10t+50 for t>2. At t=2, does the model show a continuous battery level? Is it differentiable?

WORK SPACE

ANSWER _____

2 A game designer models an avatar's position, in meters, after t seconds by $P(t)=t^2$ for t<=3 and P(t)=6t-9 for t>3. At t=3, does the avatar's position and velocity change smoothly?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Find values of a and b that make g continuous and differentiable at x=2. g(x)=ax+b for x<2, and $g(x)=x^2+1$ for x>=2.

WORK SPACE AND ANSWER

ANSWER KEY # Smooth or Split?

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	$p(x)=(x^2-1)/(x-1)$ for x!=1, and p(1)=5. Is p continuous and differentiable at x=1?	**p is neither continuous nor differentiable at x=1.**
2	q(x)=|x+2|. Is q continuous and differentiable at x=-2?	**q is continuous but not differentiable at x=-2.**
3	$r(x)=x+1$ for $x<0$, and $r(x)=x^2+1$ for $x\geq 0$. Is r continuous and differentiable at x=0?	**r is continuous but not differentiable at x=0.**
4	s(x)=1/(x-3). Is s continuous and differentiable at x=3?	**s is neither continuous nor differentiable at x=3.**
5	$t(x)=x^3-5x$. Is t continuous and differentiable at x=1?	**t is continuous and differentiable at x=1.**
6	Function m is differentiable at x=-1. What must be true about m at x=-1?	**m must be continuous at x=-1.**

PART 2 · APPLY IT

1. **The battery level is continuous but not differentiable at t=2.** Both pieces give a battery level of 70 at t=2, so there is no jump. The charging rates are 20 and 10 percentage points per hour, so the derivative changes abruptly.
2. **Yes. P is continuous and differentiable at t=3.** Both pieces give position 9 at t=3. The left velocity is 2(3)=6, and the right velocity is 6, so the velocities match.

CHALLENGE

a=4 and b=-3.

Accept equivalent statements such as "continuous, not differentiable" or "neither." For the challenge, students may use derivative matching to get a=4, then value matching to get b=-3.