

# Tangent Line Detectives

Find slopes, including vertical tangents.

HIGH SCHOOL • MA.912.C.3.1

**MA.912.C.3.1** "Find the slope of a curve at a point, including points at which there are vertical tangent lines."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## SLOPE AT A POINT

For a curve written as  $y=f(x)$ , find the slope at  $x=a$  by calculating  $f'(a)$ . For an equation with both  $x$  and  $y$ , differentiate implicitly and solve for  $dy/dx$ . If  $dy/dx$  is undefined because its denominator is 0 while its numerator is not, the tangent line is vertical.

## WORKED EXAMPLE

Find the slope of  $x=(y-1)^5+6$  at the point  $(6,1)$ .

1. Differentiate with respect to  $y$ :  
 $dx/dy=5(y-1)^4$ .
2. Substitute  $y=1$ :  $dx/dy=0$ .
3. A zero value of  $dx/dy$  means the tangent line is vertical.

**Answer: The slope is undefined. The vertical tangent line is  $x=6$ .**

## PRACTICE 6 problems

**1** Find the slope of  $y=x^3-4x+2$  at  $x=2$ .

ANSWER \_\_\_\_\_

**2** Find the slope of  $y=2x^4-3x^2$  at  $x=-1$ .

ANSWER \_\_\_\_\_

**3** Find the slope of  $x=(y+2)^3-4$  at the point  $(-4,-2)$ .

ANSWER \_\_\_\_\_

**4** Find the slope of  $x^2+y^2=25$  at the point  $(3,4)$ .

ANSWER \_\_\_\_\_

**5** Find the slope of  $y=3/x$  at  $x=-3$ .

ANSWER \_\_\_\_\_

**6** Find the slope of  $y=(x-2)^4$  at  $x=2$ .

ANSWER \_\_\_\_\_

## APPLY IT Show your work

- 1** A car's position, in meters, after  $t$  seconds is  $s(t)=t^3-8t^2+4t$ . Find the car's instantaneous velocity at  $t=2$ . Is the car momentarily stopped?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A student designs a game logo whose curved edge follows  $x=(y-2)^3+5$ . Find the slope of the edge at the point  $(5,2)$  and give the equation of its tangent line.

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

The curve  $x=y^3-3y$  has vertical tangent lines. Find both vertical tangent lines and explain how you know.

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## PART 1 • PRACTICE

| # | PROBLEM                                                  | ANSWER                                                                         |
|---|----------------------------------------------------------|--------------------------------------------------------------------------------|
| 1 | Find the slope of $y=x^3-4x+2$ at $x=2$ .                | <b>8</b>                                                                       |
| 2 | Find the slope of $y=2x^4-3x^2$ at $x=-1$ .              | <b>-2</b>                                                                      |
| 3 | Find the slope of $x=(y+2)^3-4$ at the point $(-4,-2)$ . | <b>The slope is undefined. The vertical tangent line is <math>x=-4</math>.</b> |
| 4 | Find the slope of $x^2+y^2=25$ at the point $(3,4)$ .    | <b>-3/4</b>                                                                    |
| 5 | Find the slope of $y=3/x$ at $x=-3$ .                    | <b>-1/3</b>                                                                    |
| 6 | Find the slope of $y=(x-2)^4$ at $x=2$ .                 | <b>0</b>                                                                       |

## PART 2 • APPLY IT

- 16 meters per second. No, the car is not momentarily stopped.** Differentiate to get  $s'(t)=3t^2-16t+4$ . Then  $s'(2)=12-32+4=-16$ .
- The slope is undefined. The tangent line is  $x=5$ .** Differentiate with respect to  $y$  to get  $dx/dy=3(y-2)^2$ . At  $y=2$ ,  $dx/dy=0$ , so the tangent is vertical.

## CHALLENGE

**The vertical tangent lines are  $x=-2$  and  $x=2$ . Since  $dx/dy=3y^2-3$ ,  $dx/dy=0$  when  $y=1$  or  $y=-1$ . The corresponding points are  $(-2,1)$  and  $(2,-1)$ , so the tangents are vertical at those  $x$ -values.**

Accept equivalent derivative work and equivalent statements such as "undefined slope" or "vertical tangent." For vertical tangents, students should identify the vertical line as  $x=\text{constant}$ .