Rates in Motion

Use derivatives to connect changing quantities.

HIGH SCHOOL · MA.912.C.3.10

MA.912.C.3.10 "Model and solve problems involving rates of change, including related rates."

NAME ______________________ DATE ____________

CONNECT THE CHANGES

Write an equation that connects the quantities, then differentiate both sides with respect to time. Substitute the values from the instant named in the problem only after differentiating.

WORKED EXAMPLE

A circular stain has a radius of 13 mm that is increasing at 11 mm/s. How fast is the area increasing?

1. A = pi r^2.
2. dA/dt = 2pi r(dr/dt).
3. dA/dt = 2pi(13)(11) = 286pi mm^2/s.

Answer: 286pi mm^2/s

PRACTICE 6 problems

1. The side length of a square is 8 cm and is increasing at 4 cm/s. How fast is the area increasing?

   ANSWER ____________

2. A cube has an edge length of 6 cm. Its edge length is increasing at 2 cm/s. How fast is the volume increasing?

   ANSWER ____________

3. A 17-ft ladder leans against a wall. Its bottom is 15 ft from the wall and moves away from the wall at 4 ft/s. How fast is the top of the ladder sliding down the wall?

   ANSWER ____________

4. A rectangle has length 9 cm and width 7 cm. The length is increasing at 2 cm/s, and the width is decreasing at 1 cm/s. How fast is the area changing?

   ANSWER ____________

5. A circular garden bed has a radius of 4 m that is decreasing at 3 m/s. How fast is its area changing?

   ANSWER ____________

6. A sphere has a radius of 3 cm that is increasing at 4 cm/s. How fast is its volume increasing?

   ANSWER ____________

APPLY IT Show your work

1 A drone rises straight up from a launch pad. You stand 20 m from the launch pad. When the drone is 15 m high, it rises at 12 m/s. How fast is the distance from you to the drone changing?

WORK SPACE

ANSWER ______________________

2 In an online game, an avatar is 9 units east and 12 units north of a checkpoint. It is moving east at 4 units/s and north at 3 units/s. How fast is its distance from the checkpoint changing?

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

A cone always has a radius equal to half its height. At an instant when its height is 10 cm, the height is increasing at 3 cm/s. How fast is the cone's volume increasing?

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The side length of a square is 8 cm and is increasing at 4 cm/s. How fast is the area increasing?	**64 cm^2/s**
2	A cube has an edge length of 6 cm. Its edge length is increasing at 2 cm/s. How fast is the volume increasing?	**216 cm^3/s**
3	A 17-ft ladder leans against a wall. Its bottom is 15 ft from the wall and moves away from the wall at 4 ft/s. How fast is the top of the ladder sliding down the wall?	**15/2 ft/s downward**
4	A rectangle has length 9 cm and width 7 cm. The length is increasing at 2 cm/s, and the width is decreasing at 1 cm/s. How fast is the area changing?	**5 cm^2/s**
5	A circular garden bed has a radius of 4 m that is decreasing at 3 m/s. How fast is its area changing?	**-24pi m^2/s**
6	A sphere has a radius of 3 cm that is increasing at 4 cm/s. How fast is its volume increasing?	**144pi cm^3/s**

PART 2 · APPLY IT

1. **36/5 m/s** Use d^2 = 20^2 + h^2. At h = 15, d = 25, so 2d(d') = 2h(h'), giving d' = 36/5 m/s.
2. **24/5 units/s** Use d^2 = x^2 + y^2. Since d = 15, d' = (x x' + y y')/d = (9(4) + 12(3))/15 = 24/5 units/s.

CHALLENGE

75pi cm^3/s

Accept equivalent exact forms with pi. For problem 3, accept -15/2 ft/s if the student defines upward as positive, and for problem 5, accept “decreasing at 24pi m^2/s.”