

Tangent Line Toolkit

Build tangent equations and use them for nearby estimates.

HIGH SCHOOL • MA.912.C.3.2

MA.912.C.3.2 "Find an equation for the tangent line to a curve at a point and use it to make local linear approximation."

NAME _____

DATE _____

A NEARBY LINE

At $x = a$, the tangent line has slope $f'(a)$ and passes through $(a, f(a))$. Write the local linear model as $L(x) = f(a) + f'(a)(x - a)$, then use $L(x)$ to estimate $f(x)$ for x -values close to a .

WORKED EXAMPLE

For $f(x) = x^2$, find the tangent line at $x = 4$. Then use it to estimate $f(17/4)$.

1. $f(4) = 16$, so the point is $(4, 16)$.
2. $f'(x) = 2x$, so $f'(4) = 8$.
3. Use point-slope form: $y - 16 = 8(x - 4)$.
4. $L(17/4) = 16 + 8(17/4 - 4) = 18$.

**Answer: Tangent line: $y - 16 = 8(x - 4)$.
Estimate: $f(17/4)$ is about 18.**

PRACTICE 6 problems

- 1** For $f(x) = x^3$, find the equation of the tangent line at $x = 3$.

ANSWER _____

- 2** For $f(x) = \sqrt{x}$, find the equation of the tangent line at $x = 9$.

ANSWER _____

- 3** For $f(x) = 1/x$, find the equation of the tangent line at $x = 3$.

ANSWER _____

- 4** For $f(x) = x^2 + 3x$, find the equation of the tangent line at $x = -1$.

ANSWER _____

- 5** Use a local linear approximation for $f(x) = x^{1/3}$ at $x = 27$ to estimate cube root of 28.

ANSWER _____

- 6** Use a local linear approximation for $f(x) = \sqrt{x}$ at $x = 9$ to estimate $\sqrt{10}$.

ANSWER _____

APPLY IT Show your work

- 1** A drone club models a drone's height, in meters, with $h(x) = x^2 + 5x + 1$, where x is time in seconds. Find the tangent line at $x = 2$, then use it to estimate the height at $x = 21/10$ seconds.

WORK SPACE

ANSWER _____

- 2** The revenue from a school concert is modeled by $R(x) = 100x - x^2$, where x is the ticket price in dollars and $R(x)$ is revenue in dollars. Find the tangent line at $x = 40$, then use it to estimate the revenue when tickets cost \$41.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $f(x) = x^4$, find the tangent line at $x = 3$ and use it to estimate $f(31/10)$. Explain whether your estimate is too low or too high.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x^3$, find the equation of the tangent line at $x = 3$.	$y - 27 = 27(x - 3)$
2	For $f(x) = \sqrt{x}$, find the equation of the tangent line at $x = 9$.	$y - 3 = \frac{1}{6}(x - 9)$
3	For $f(x) = 1/x$, find the equation of the tangent line at $x = 3$.	$y - \frac{1}{3} = -\frac{1}{9}(x - 3)$
4	For $f(x) = x^2 + 3x$, find the equation of the tangent line at $x = -1$.	$y + 2 = x + 1$
5	Use a local linear approximation for $f(x) = x^{1/3}$ at $x = 27$ to estimate cube root of 28.	cube root of 28 is about $\frac{82}{27}$
6	Use a local linear approximation for $f(x) = \sqrt{x}$ at $x = 9$ to estimate $\sqrt{10}$.	$\sqrt{10}$ is about $\frac{19}{6}$

PART 2 • APPLY IT

- Tangent line: $y - 15 = 9(x - 2)$. Estimated height: $h(21/10)$ is about $159/10$ meters.** $h(2) = 15$ and $h'(2) = 9$, so use $L(x) = 15 + 9(x - 2)$. Evaluating $L(21/10)$ gives $159/10$.
- Tangent line: $y - 2400 = 20(x - 40)$. Estimated revenue: $R(41)$ is about \$2420.** $R(40) = 2400$ and $R'(40) = 20$, so use $L(x) = 2400 + 20(x - 40)$. Then $L(41) = 2420$.

CHALLENGE

Tangent line: $y - 81 = 108(x - 3)$. $f(31/10)$ is about $459/5$. The estimate is too low because $f(x) = x^4$ is concave up near $x = 3$, so its tangent line lies below the curve there.

Accept equivalent tangent-line forms, including slope-intercept form. Accept equivalent fractions or terminating decimals for local linear estimates.